

Grade Three

STANDARD 4.1 (NUMBER AND NUMERICAL OPERATIONS) ALL STUDENTS WILL DEVELOP NUMBER SENSE AND WILL PERFORM STANDARD NUMERICAL OPERATIONS AND ESTIMATIONS ON ALL TYPES OF NUMBERS IN A VARIETY OF WAYS.

Descriptive Statement: Numbers and arithmetic operations are what most of the general public think about when they think of mathematics; and, even though other areas like geometry, algebra, and data analysis have become increasingly important in recent years, numbers and operations remain at the heart of mathematical teaching and learning. Facility with numbers, the ability to choose the appropriate types of numbers and the appropriate operations for a given situation, and the ability to perform those operations as well as to estimate their results, are all skills that are essential for modern day life.

Number Sense. Number sense is an intuitive feel for numbers and a common sense approach to using them. It is a comfort with what numbers represent that comes from investigating their characteristics and using them in diverse situations. It involves an understanding of how different types of numbers, such as fractions and decimals, are related to each other, and how each can best be used to describe a particular situation. It subsumes the more traditional category of school mathematics curriculum called numeration and thus includes the important concepts of place value, number base, magnitude, and approximation and estimation.

Numerical Operations. Numerical operations are an essential part of the mathematics curriculum, especially in the elementary grades. Students must be able to select and apply various computational methods, including mental math, pencil-and-paper techniques, and the use of calculators. Students must understand how to add, subtract, multiply, and divide whole numbers, fractions, decimals, and other kinds of numbers. With the availability of calculators that perform these operations quickly and accurately, the instructional emphasis now is on understanding the meanings and uses of these operations, and on estimation and mental skills, rather than solely on the development of paper-and-pencil proficiency.

Estimation. Estimation is a process that is used constantly by mathematically capable adults, and one that can be easily mastered by children. It involves an educated guess about a quantity or an intelligent prediction of the outcome of a computation. The growing use of calculators makes it more important than ever that students know when a computed answer is reasonable; the best way to make that determination is through the use of strong estimation skills. Equally important is an awareness of the many situations in which an approximate answer is as good as, or even preferable to, an exact one. Students can learn to make these judgments and use mathematics more powerfully as a result.

Number and operation skills continue to be a critical piece of the school mathematics curriculum and, indeed, a very important part of mathematics. But, there is perhaps a greater need for us to rethink our approach here than to do so for any other curriculum component. An enlightened mathematics program for today's children will empower them to use all of today's tools rather than require them to meet yesterday's expectations.

Building upon knowledge and skills gained in preceding grades,
by the end of **Grade 3**, students will:

NUMBER SENSE	<input type="checkbox"/> Use real-life experiences, physical materials, and technology to construct meanings for number experiences and problems
	<input type="checkbox"/> Demonstrate an understanding of whole number place value concepts: 10 ones = 1 ten; 10 tens = 1 hundred; 10 hundreds = 1 thousand
	<input type="checkbox"/> Identify place value of thousands to hundred thousand
	<input type="checkbox"/> Explore the extension of the place value system to decimals through hundredths.
	<input type="checkbox"/> Explore the meaning of whole numbers through hundred thousands
	<input type="checkbox"/> Skip count by 25, 50, 100 up to 1000
	<input type="checkbox"/> Read and write whole numbers up to 1,000
<input type="checkbox"/> Compare and order numbers to 1,000	
<input type="checkbox"/> Identify whether any whole number is odd or even .	
<input type="checkbox"/> Understand the various uses of numbers. Counting, measuring, labeling (e.g., numbers on baseball uniforms)	
	<input type="checkbox"/> Read and write ordinal numbers first to fiftieth

ADDITION AND SUBTRACTION	<input type="checkbox"/> Relate addition to subtraction; subtraction to addition
MULTIPLICATION AND DIVISION	<input type="checkbox"/> Explore the meaning of multiplication through child centered activities <input type="checkbox"/> Relate multiplication to addition <input type="checkbox"/> Discover and explain what happens when 1 and 0 are factors of multiplication <input type="checkbox"/> Discover and explain what happens when 1 and 0 are factors of multiplication <input type="checkbox"/> Understand division using concrete materials on real world – child centered - problems <input type="checkbox"/> Show division as a way of sharing equally <input type="checkbox"/> Demonstrate division by using manipulatives <input type="checkbox"/> Use tables, patterns, halving, and lots of manipulatives to provide meaning for division
FRACTIONS	<input type="checkbox"/> Identify fractions as part of a whole using concrete materials with child centered activities <input type="checkbox"/> Compare common equivalent fractions using concrete and representative material <input type="checkbox"/> Recognize, read and write fractional parts of a group whose denominators are: 2, 3, 4, 5, 6, 8, and 10 $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$) <input type="checkbox"/> Understand and recognize the meaning and word for numerator and denominator . <input type="checkbox"/> Explore equivalent fractions ($\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$)
MONEY	<input type="checkbox"/> Compare amounts; make change <input type="checkbox"/> Add money <input type="checkbox"/> Subtract money
DECIMALS	<input type="checkbox"/> Explore equivalent money value - \$1.00 = 10 dimes; 20 nickels, 100 pennies; 2 half dollars...

NUMERICAL OPERATION

ADDITION AND SUBTRACTION	<input type="checkbox"/> Perform two and three digit addition with regrouping <input type="checkbox"/> Perform two and three digit addition without regrouping <input type="checkbox"/> Perform two and three digit subtraction with regrouping <input type="checkbox"/> Perform two and three digit subtraction without regrouping <input type="checkbox"/> Subtract three digit numbers with regrouping across zeros <input type="checkbox"/> Add and subtract money to three digits
MULTIPLICATION AND DIVISION	<input type="checkbox"/> Multiply two by one digit (24x3) <input type="checkbox"/> Know with accuracy and speed – by the end of Grade the - multiplication facts to 9

ESTIMATION	<input type="checkbox"/> Judge without counting whether a set of objects has less than, more than, or the same number of objects as a reference set.
	<input type="checkbox"/> Use concrete and semi-concrete activities to estimate quantities of 10, 100, 1000
	<input type="checkbox"/> Estimate numbers up to 500
	<input type="checkbox"/> Construct and use a variety of estimation strategies (e.g., rounding and mental math) for estimating both quantities and the result of computations.
	<input type="checkbox"/> Recognize when an estimate is appropriate, and understand the usefulness of an estimate as distinct from an exact answer.



STANDARD 4.2 (GEOMETRY AND MEASUREMENT) ALL STUDENTS WILL DEVELOP SPATIAL SENSE AND THE ABILITY TO USE GEOMETRIC PROPERTIES, RELATIONSHIPS, AND MEASUREMENT TO MODEL, DESCRIBE AND ANALYZE PHENOMENA.

Descriptive Statement: Spatial sense is an intuitive feel for shape and space. Geometry and measurement both involve describing the shapes we see all around us in art, nature, and the things we make. Spatial sense, geometric modeling, and measurement can help us to describe and interpret our physical environment and to solve problems.

Geometric Properties. This includes identifying, describing and classifying standard geometric objects, describing and comparing properties of geometric objects, making conjectures concerning them, and using reasoning and proof to verify or refute conjectures and theorems. Also included here are such concepts as symmetry, congruence, and similarity.

Transforming Shapes. Analyzing how various transformations affect geometric objects allows students to enhance their spatial sense. This includes combining shapes to form new ones and decomposing complex shapes into simpler ones. It includes the standard geometric transformations of translation (slide), reflection (flip), rotation (turn), and dilation (scaling). It also includes using tessellations and fractals to create geometric patterns.

Coordinate Geometry. Coordinate geometry provides an important connection between geometry and algebra. It facilitates the visualization of algebraic relationships, as well as an analytical understanding of geometry.

Units of Measurement. Measurement helps describe our world using numbers. An understanding of how we attach numbers to real-world phenomena, familiarity with common measurement units (e.g., inches, liters, and miles per hour), and a practical knowledge of measurement tools and techniques are critical for students' understanding of the world around them.

Measuring Geometric Objects. This area focuses on applying the knowledge and understandings of units of measurement in order to actually perform measurement. While students will eventually apply formulas, it is important that they develop and apply strategies that derive from their understanding of the attributes. In addition to measuring objects directly, students apply indirect measurement skills, using, for example, similar triangles and trigonometry.

Students of all ages should realize that geometry and measurement is all around them. Through study of these areas and their applications, they should come to better understand and appreciate the role of mathematics in their lives.

Building upon knowledge and skills gained in preceding grades, by the end of **Grade 3**, students will:

<p>GEOMETRIC PROPERTIES</p>	<ul style="list-style-type: none"> <input type="checkbox"/> Identify and describe spatial relationships of two or more objects in space <input type="checkbox"/> Use concrete materials to show: Direction, orientation, and perspectives (e.g., which object is on your left when you are standing here) <input type="checkbox"/> Compare and contrast relative shapes and sizes <input type="checkbox"/> Identify and describe relationships among two and three dimensional shapes <hr/> <ul style="list-style-type: none"> <input type="checkbox"/> Use properties of standard two -dimensional and three-dimensional shapes to identify, classify and describe the shapes by: <ol style="list-style-type: none"> 1. Their parts: Vertex, edge, face, side, angle 2. 2D figures – square, rectangle, circle, triangle, pentagon, hexagon, octagon 3. 3D figures – cube, rectangular prism, sphere, cone, cylinder, and pyramid <input type="checkbox"/> Identify, describe and study the relationship among two dimensional shapes: <ol style="list-style-type: none"> 1. Same size; same shape 2. Lines of symmetry <input type="checkbox"/> Understand and apply concepts involving lines, angles, and circles Line, line segment, endpoint
<p>TRANSFORMING SHAPES</p>	<ul style="list-style-type: none"> <input type="checkbox"/> Describe and use geometric transformations (slide, flip, turn). <input type="checkbox"/> Investigate the occurrence of geometry in nature and art.
<p>COORDINATE GEOMETRY</p>	<ul style="list-style-type: none"> <input type="checkbox"/> Locate and name points in the first quadrant on a coordinate grid.
<p>UNITS OF MEASUREMENT</p>	<ul style="list-style-type: none"> <input type="checkbox"/> Understand that everyday objects have a variety of attributes, each of which can be measured in many ways. <input type="checkbox"/> Select and use appropriate standard units of measure and measurement tools to solve real-life problems. <input type="checkbox"/> Incorporate estimation in measurement activities (e.g., estimate before measuring).

LENGTH Do all measures live: real objects, real measures	<input type="checkbox"/> Measure length of a body part (foot, hand) and use this to determine actual measurements of various items <input type="checkbox"/> Length – fraction of an inch $\frac{1}{2}$, $\frac{1}{4}$ <input type="checkbox"/> Area – square inch, square centimeter <input type="checkbox"/> Measure various lengths in inches, feet, yards <input type="checkbox"/> Measure various lengths to nearest centimeters and meters
CAPACITY	<input type="checkbox"/> Measure fluid ounce , cup, gallon, <input type="checkbox"/> Measure liquids in milliliter
MASS	<input type="checkbox"/> Measure food and drink in grams, kilograms
WEIGHT	<input type="checkbox"/> Measure weight in ounces
MONEY	<input type="checkbox"/> Express money values using appropriate decimal terms
TIME	<input type="checkbox"/> Determine elapsed time given start and finish to nearest hour during same twelve hour period <input type="checkbox"/> Read time on an analog clock to the nearest minute
TEMPERATURE	<input type="checkbox"/> Read thermometer in Fahrenheit and Celsius to nearest degree
ESTIMATION	<input type="checkbox"/> Estimate weight in ounces, <input type="checkbox"/> Estimate capacity using cups, gallons, millimeters <input type="checkbox"/> Estimate length, width, height, volume <input type="checkbox"/> Estimate elapsed time

Special Note: All measurement must be done Live in order to make sense to the students. Use real measures, real objects to do your measurement.

MEASURING GEOMETRIC OBJECTS	<input type="checkbox"/> Find area of objects using one inch squares and centimeter squares on graph paper <input type="checkbox"/> Determine the area of simple 2D shapes on a square grid . <input type="checkbox"/> Use a ruler to measure the perimeter of objects. Note: Yarn is great for measuring larger objects!
	<input type="checkbox"/> Measure and compare the volume of 3D objects using materials such as rice or cubes.



STANDARD 4.3 (PATTERNS AND ALGEBRA) ALL STUDENTS WILL REPRESENT AND ANALYZE RELATIONSHIPS AMONG VARIABLE QUANTITIES AND SOLVE PROBLEMS INVOLVING PATTERNS, FUNCTIONS, AND ALGEBRAIC CONCEPTS AND PROCESSES.

Descriptive Statement: Algebra is a symbolic language used to express mathematical relationships. Students need to understand how quantities are related to one another, and how algebra can be used to concisely express and analyze those relationships. Modern technology provides tools for supplementing the traditional focus on algebraic procedures, such as solving equations, with a more visual perspective, with graphs of equations displayed on a screen. Students can then focus on understanding the relationship between the equation and the graph, and on what the graph represents in a real-life situation.

Patterns. Algebra provides the language through which we communicate the patterns in mathematics. From the earliest age, students should be encouraged to investigate the patterns that they find in numbers, shapes, and expressions, and, by doing so, to make mathematical discoveries. They should have opportunities to analyze, extend, and create a variety of patterns and to use pattern-based thinking to understand and represent mathematical and other real-world phenomena.

Functions and Relationships. The function concept is one of the most fundamental unifying ideas of modern mathematics. Students begin their study of functions in the primary grades, as they observe and study patterns. As students grow and their ability to abstract matures, students form rules, display information in a table or chart, and write equations, which express the relationships they have observed. In high school, they use the more formal language of algebra to describe these relationships.

Modeling. Algebra is used to model real situations and answer questions about them. This use of algebra requires the ability to represent data in tables, pictures, graphs, equations or inequalities, and rules. Modeling ranges from writing simple number sentences to help solve story problems in the primary grades to using functions to describe the relationship between two variables, such as the height of a pitched ball over time. Modeling also includes some of the conceptual building blocks of calculus, such as how quantities change over time and what happens in the long run (limits).

Procedures. Techniques for manipulating algebraic expressions – procedures – remain important, especially for students who may continue their study of mathematics in a calculus program. Utilization of algebraic procedures includes understanding and applying properties of numbers and operations, using symbols and variables appropriately, working with expressions, equations, and inequalities, and solving equations and inequalities.

Algebra is a gatekeeper for the future study of mathematics, science, the social sciences, business, and a host of other areas. In the past, algebra has served as a filter, screening people out of these opportunities. For New Jersey to be part of the global society, it is important that algebra play a major role in a mathematics program that opens the gates for all students.

Building upon knowledge and skills gained in preceding grades,
By the end of **Grade 3**, students will:

PATTERNS	<input type="checkbox"/> Recognize, describe, extend, and create patterns using concrete materials, pictures, rhythms, whole numbers....
	<input type="checkbox"/> Describe patterns using words and symbols (e.g. add two....)
	<input type="checkbox"/> Discover whole number patterns that grow or shrink as a result of repeatedly adding, subtracting, multiplying by, or dividing by a fixed number (e.g. 5, 8, 11...or 800,400,200...)

FUNCTIONS AND RELATIONSHIPS	<input type="checkbox"/> Use concrete and pictorial models to explore the basic concept of a function. Use Input/output tables, T-charts
	<input type="checkbox"/> Represent unknown quantities using symbols, boxes, pictures, blanks $27 + \underline{\quad} = 35$
	<input type="checkbox"/> Explore substitution a number value for letters to find the sum, using concrete and semi-concrete activities ($3+a= \underline{\quad}$) if $a =2$; if $a =3$

MODELING	<input type="checkbox"/> Recognize and describe change in quantities....from graphs showing temperature, height...
	<input type="checkbox"/> Construct and solve simple open sentences involving addition or subtraction ($3 + 6 = \underline{\quad}$) $N = 15-3$ $16 - c = 7$

PROCEDURES	<input type="checkbox"/> Understand and apply the properties of operations and numbers Commutative (e.g., $3 \times 7 = 7 \times 3$)
	<input type="checkbox"/> Associative (e.g. $7+3+2$ can be found by first adding either $7+3$ or $3+2$)
	<input type="checkbox"/> Identify element for multiplication is 1 (e.g. $1 \times 8 = 8$)
	<input type="checkbox"/> Any number multiplied by zero is zero ($2 \times 0 = 0$)
	<input type="checkbox"/> Understand and use the concepts of equals, less than, and greater than to describe relations
	<input type="checkbox"/> Compare numbers using symbols ($=, <, >$)

STANDARD 4.4 (DATA ANALYSIS, PROBABILITY, AND DISCRETE MATHEMATICS) ALL STUDENTS WILL DEVELOP AN UNDERSTANDING OF THE CONCEPTS AND TECHNIQUES OF DATA ANALYSIS, PROBABILITY, AND DISCRETE MATHEMATICS, AND WILL USE THEM TO MODEL SITUATIONS, SOLVE PROBLEMS, AND ANALYZE AND DRAW APPROPRIATE INFERENCES FROM DATA.

Descriptive Statement: Data analysis, probability, and discrete mathematics are important interrelated areas of applied mathematics. Each provides students with powerful mathematical perspectives on everyday phenomena and with important examples of how mathematics is used in the modern world. Two important areas of discrete mathematics are addressed in this standard; a third area, iteration and recursion, is addressed in Standard 4.3 (Patterns and Algebra).

Data Analysis (or Statistics). In today's information-based world, students need to be able to read, understand, and interpret data in order to make informed decisions. In the early grades, students should be involved in collecting and organizing data, and in presenting it using tables, charts, and graphs. As they progress, they should gather data using sampling, and should increasingly be expected to analyze and make inferences from data, as well as to analyze data and inferences made by others.

Probability. Students need to understand the fundamental concepts of probability so that they can interpret weather forecasts, avoid unfair games of chance, and make informed decisions about medical treatments whose success rate is provided in terms of percentages. They should regularly be engaged in predicting and determining probabilities, often based on experiments (like flipping a coin 100 times), but eventually based on theoretical discussions of probability that make use of systematic counting strategies. High school students should use probability models and solve problems involving compound events and sampling.

Discrete Mathematics—Systematic Listing and Counting. Development of strategies for listing and counting can progress through all grade levels, with middle and high school students using the strategies to solve problems in probability. Primary students, for example, might find all outfits that can be worn using two coats and three hats; middle school students might systematically list and count the number of routes from one site on a map to another; and high school students might determine the number of three-person delegations that can be selected from their class to visit the mayor.

Discrete Mathematics—Vertex-Edge Graphs and Algorithms. Vertex-edge graphs, consisting of dots (vertices) and lines joining them (edges), can be used to represent and solve problems based on real-world situations. Students should learn to follow and devise lists of instructions, called "algorithms," and use algorithmic thinking to find the best solution to problems like those involving vertex-edge graphs, but also to solve other problems.

These topics provide students with insight into how mathematics is used by decision-makers in our society, and with important tools for modeling a variety of real-world situations. Students will better understand and interpret the vast amounts of quantitative data that they are exposed to daily, and they will be able to judge the validity of data-supported arguments.

Building upon knowledge and skills gained in preceding grades, by the end of **Grade 3**, students will:

<p>DATA ANALYSIS</p>	<ul style="list-style-type: none"> <input type="checkbox"/> Collect, generate, organize, and display data in response to questions, claims, or curiosity. Collect data from the classroom environment <input type="checkbox"/> Read, interpret, construct, analyze, generate questions about, and draw inferences from displays of data. <input type="checkbox"/> Read, generate pictograph, bar graph, table
<p>PROBABILITY</p>	<ul style="list-style-type: none"> <input type="checkbox"/> Use everyday events and chance devices, such as dice, coins, and unevenly divided spinners, to explore concepts of probability: Likely, unlikely, certain, possible, impossible, more likely, less likely, equally likely <input type="checkbox"/> Predict probabilities in a variety of situations (e.g. given the number of items of each color in a bag, what is the probability that an item picked will have a particular color?) <input type="checkbox"/> Notice what students think will happen? (intuitive skills) <input type="checkbox"/> Collect data and use that data to predict the probability (experimental)
<p>DISCRETE MATHEMATICS – SYSTEMATIC LISTING AND COUNTING</p>	<ul style="list-style-type: none"> <input type="checkbox"/> Represent and classify data according to attributes, such as shape or color, and relationships: <input type="checkbox"/> Create Venn diagrams based on math data <input type="checkbox"/> Put digits in numerical order <input type="checkbox"/> Represent all possibilities for a simple counting situation in an organized way and draw conclusions from this representation: <input type="checkbox"/> Create and organized lists, charts on real world data
<p>DISCRETE MATHEMATICS – VERTEX-EDGE GRAPHS AND ALGORITHMS</p>	<ul style="list-style-type: none"> <input type="checkbox"/> Follow, devise, and describe practical sets of directions (e.g., to add two 2-digit numbers). <input type="checkbox"/> Explore vertex-edge graphs: Note the vertex, edge; follow the path <input type="checkbox"/> Find the smallest number of colors needed to color a map.

STANDARD 4.5 (MATHEMATICAL PROCESSES) ALL STUDENTS WILL USE MATHEMATICAL PROCESSES OF PROBLEM SOLVING, COMMUNICATION, CONNECTIONS, REASONING, REPRESENTATIONS, AND TECHNOLOGY TO SOLVE PROBLEMS AND COMMUNICATE MATHEMATICAL IDEAS.

Descriptive Statement: The mathematical processes described here highlight ways of acquiring and using the content knowledge and skills delineated in the first four mathematics standards.

Problem Solving. Problem posing and problem solving involve examining situations that arise in mathematics and other disciplines and in common experiences, describing these situations mathematically, formulating appropriate mathematical questions, and using a variety of strategies to find solutions. Through problem solving, students experience the power and usefulness of mathematics. Problem solving is interwoven throughout the grades to provide a context for learning and applying mathematical ideas.

Communication. Communication of mathematical ideas involves students' sharing their mathematical understandings in oral and written form with their classmates, teachers, and parents. Such communication helps students clarify and solidify their understanding of mathematics and develop confidence in them as mathematics learners. It also enables teachers to better monitor student progress.

Connections. Making connections involves seeing relationships between different topics, and drawing on those relationships in future study. This applies within mathematics, so that students can translate readily between fractions and decimals, or between algebra and geometry; to other content areas, so that students understand how mathematics is used in the sciences, the social sciences, and the arts; and to the everyday world, so that students can connect school mathematics to daily life.

Reasoning. Mathematical reasoning is the critical skill that enables a student to make use of all other mathematical skills. With the development of mathematical reasoning, students recognize that mathematics makes sense and can be understood. They learn how to evaluate situations, select problem-solving strategies, draw logical conclusions, develop and describe solutions, and recognize how those solutions can be applied.

Representations. Representations refer to the use of physical objects, drawings, charts, graphs, and symbols to represent mathematical concepts and problem situations. By using various representations, students will be better able to communicate their thinking and solve problems. Using multiple representations will enrich the problem solver with alternative perspectives on the problem. Historically, people have developed and successfully used manipulatives (concrete representations such as fingers, base ten blocks, Geoboards, and algebra tiles) and other representations (such as coordinate systems) to help them understand and develop mathematics.

Technology. Calculators and computers need to be used along with other mathematical tools by students in both instructional and assessment activities. These tools should be used, not to replace mental math and paper-and-pencil computational skills, but to enhance understanding of mathematics and the power to use mathematics. Students should explore both new and familiar concepts with calculators and computers and should also become proficient in using technology as it is used by adults (e.g., for assistance in solving real-world problems).

At each grade level, with respect to content appropriate for that grade level, students will:

PROBLEM SOLVING

- Learn mathematics through problem solving, inquiry, and discovery.
- Solve problems that arise in mathematics and in other contexts (cf. workplace readiness standard 8.3).
 - Open-ended problems
 - Non-routine problems
 - Problems with multiple solutions
 - Problems that can be solved in several ways
- Select and apply a variety of appropriate problem-solving strategies (e.g., “try a simpler problem” or “make a diagram”) to solve problems.
- Pose problems of various types and levels of difficulty.
- Monitor their progress and reflect on the process of their problem solving activity.

COMMUNICATION

- Use communication to organize and clarify their mathematical thinking.
 - Reading and writing, discussions, listening, and questioning
- Communicate their mathematical thinking coherently and clearly to peers, teachers, and others, both orally and in writing.
- Analyze and evaluate the mathematical thinking and strategies of others.
- Use the language of mathematics to express mathematical ideas precisely.

CONNECTIONS

- Recognize recurring themes across mathematical domains (e.g., patterns in number, algebra, and geometry).
- Use connections among mathematical ideas to explain concepts (e.g., two linear equations have a unique solution because the lines they represent intersect at a single point).
- Recognize that mathematics is used in a variety of contexts outside of mathematics.
- Apply mathematics in practical situations and in other disciplines.
- Trace the development of mathematical concepts over time and across cultures (cf. world languages and social studies standards).
- Understand how mathematical ideas interconnect and build on one another to produce a coherent whole.

REASONING

- Recognize that mathematical facts, procedures, and claims must be justified.
- Use reasoning to support their mathematical conclusions and problem solutions.
- Select and use various types of reasoning and methods of proof.
- Rely on reasoning, rather than answer keys, teachers, or peers, to check the correctness of their problem solutions.
- Make and investigate mathematical conjectures.
- Counterexamples as a means of disproving conjectures
- Verifying conjectures using informal reasoning or proofs.
- Evaluate examples of mathematical reasoning and determine whether they are valid.

REPRESENTATIONS

- Create and use representations to organize, record, and communicate mathematical ideas.
- Concrete representations (e.g., base-ten blocks or algebra tiles)
- Pictorial representations (e.g., diagrams, charts, or tables)
- Symbolic representations (e.g., a formula)
- Graphical representations (e.g., a line graph)
- Select, apply, and translate among mathematical representations to solve problems.
- Use representation to model and interpret physical, social, and mathematical phenomena.

TECHNOLOGY

- Use technology to gather, analyze, and communicate mathematical information.
- Use computer spreadsheets, software, and graphing utilities to organize and display quantitative information (cf. workplace readiness standard 8.4-D).
- Use graphing calculators and computer software to investigate properties of functions and their graphs.
- Use calculators as problem-solving tools (e.g., to explore patterns, to validate solutions).
- Use computer software to make and verify conjectures about geometric objects.
- Use computer-based laboratory technology for mathematical applications in the sciences (cf. science standards)