

GRADE EIGHT

STANDARD 4.1 (NUMBER AND NUMERICAL OPERATIONS) ALL STUDENTS WILL DEVELOP NUMBER SENSE AND WILL PERFORM STANDARD NUMERICAL OPERATIONS AND ESTIMATIONS ON ALL TYPES OF NUMBERS IN A VARIETY OF WAYS.

Descriptive Statement: Numbers and arithmetic operations are what most of the general public think about when they think of mathematics; and, even though other areas like geometry, algebra, and data analysis have become increasingly important in recent years, numbers and operations remain at the heart of mathematical teaching and learning. Facility with numbers, the ability to choose the appropriate types of numbers and the appropriate operations for a given situation, and the ability to perform those operations as well as to estimate their results, are all skills that are essential for modern day life.

Number Sense. Number sense is an intuitive feel for numbers and a common sense approach to using them. It is a comfort with what numbers represent that comes from investigating their characteristics and using them in diverse situations. It involves an understanding of how different types of numbers, such as fractions and decimals, are related to each other, and how each can best be used to describe a particular situation. It subsumes the more traditional category of school mathematics curriculum called numeration and thus includes the important concepts of place value, number base, magnitude, and approximation and estimation.

Numerical Operations. Numerical operations are an essential part of the mathematics curriculum, especially in the elementary grades. Students must be able to select and apply various computational methods, including mental math, pencil-and-paper techniques, and the use of calculators. Students must understand how to add, subtract, multiply, and divide whole numbers, fractions, decimals, and other kinds of numbers. With the availability of calculators that perform these operations quickly and accurately, the instructional emphasis now is on understanding the meanings and uses of these operations, and on estimation and mental skills, rather than solely on the development of paper-and-pencil proficiency.

Estimation. Estimation is a process that is used constantly by mathematically capable adults, and one that can be easily mastered by children. It involves an educated guess about a quantity or an intelligent prediction of the outcome of a computation. The growing use of calculators makes it more important than ever that students know when a computed answer is reasonable; the best way to make that determination is through the use of strong estimation skills. Equally important is an awareness of the many situations in which an approximate answer is as good as, or even preferable to, an exact one. Students can learn to make these judgments and use mathematics more powerfully as a result.

Number and operation skills continue to be a critical piece of the school mathematics curriculum and, indeed, a very important part of mathematics. But, there is perhaps a greater need for us to rethink our approach here than to do so for any other curriculum component. An enlightened mathematics program for today's children will empower them to use all of today's tools rather than require them to meet yesterday's expectations.

Building upon knowledge and skills gained in preceding grades, by the end of **Grade 8**, students will:

NUMBER SENSE

NUMBER SENSE	<ul style="list-style-type: none"><input type="checkbox"/> Extend understanding of the number system by constructing meanings for the following:<ul style="list-style-type: none"><input type="checkbox"/> Rational numbers<input type="checkbox"/> Percents<input type="checkbox"/> Whole numbers with exponents<input type="checkbox"/> Roots<input type="checkbox"/> Absolute values<input type="checkbox"/> Numbers represented in scientific notation <input type="checkbox"/> Compare and order numbers of all named types<input type="checkbox"/> Rewrite numbers from standard to scientific notation<input type="checkbox"/> Show the relationship of scientific notation to place value using positive and negative numbers and integer<input type="checkbox"/> Use whole numbers, fractions, decimals, and percents to represent equivalent forms of the same number<input type="checkbox"/> Recognize that repeating decimals correspond to fractions and determine their fractional equivalents $\frac{5}{7} = 0.714285714285 = \text{-----}$<p style="text-align: center;">0.714285</p><input type="checkbox"/> Construct meanings for common irrational numbers such as π pi and the square root of 2
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NUMERICAL OPERATIONS

NUMERICAL OPERATIONS	<p>Note: Students must be able to add, subtract, multiply and divide whole numbers with accuracy and speed. Maintenance matters with basic math operations. Always include basic math problems on any quiz or exam.</p> <ul style="list-style-type: none"><input type="checkbox"/> Do lots of mental math as a warm up to the day's lessons.<input type="checkbox"/> Use exponentiation to find whole number powers of numbers.<input type="checkbox"/> Find square and cube roots of numbers and understand the inverse nature of powers and roots.<input type="checkbox"/> Do square root estimates<input type="checkbox"/> Multiple and divide using scientific notation<input type="checkbox"/> Find the value of numbers written in exponential form
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FRACTIONS	<input type="checkbox"/> Review and constantly maintain all fractions – and all operations - with the fractions: Proper, improper, mixed; add, subtract, multiply and divide Note: Students must have be able to do fractions with accuracy and speed in order to move further into math concepts
DECIMALS	<input type="checkbox"/> Review and constantly maintain all decimals – and all operations with the decimals Note: Students must have be able to do decimals with accuracy and speed in order to move further into math concepts
RATIO AND PROPORTION	<input type="checkbox"/> Solve problems involving proportions and percents . <input type="checkbox"/> Use direct, inverse and partitive proportions <input type="checkbox"/> Relate proportions and ratios
PERCENT	<input type="checkbox"/> Solve real world problems from the students' world involving percents Find: <ul style="list-style-type: none"> <input type="checkbox"/> Simple and compound interest, <input type="checkbox"/> Commission, profit and loss, <input type="checkbox"/> Charges on credit card balance <input type="checkbox"/> Income tax <input type="checkbox"/> Solve real-life applications from the students' world: <ul style="list-style-type: none"> <input type="checkbox"/> Percent <input type="checkbox"/> Gratuities <input type="checkbox"/> Discount <input type="checkbox"/> Simple and compound interest

ESTIMATION

ESTIMATION	<input type="checkbox"/> Estimate square and cube roots of numbers. <input type="checkbox"/> Use equivalent representations of numbers such as fractions, decimals, and percents to facilitate estimation. <input type="checkbox"/> Recognize the limitations of estimation and assess the amount of error resulting from estimation. <input type="checkbox"/> Estimate answers to computational type problems to determine reasonableness of answer <input type="checkbox"/> Estimate percentages of numbers <input type="checkbox"/> Use equivalent numbers to make estimation easier (e.g. fractions to decimals to percents) <input type="checkbox"/> Determine and explain rational numbers, whether to overestimate or underestimate, in problem solving activities
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STANDARD 4.2 (GEOMETRY AND MEASUREMENT) ALL STUDENTS WILL DEVELOP SPATIAL SENSE AND THE ABILITY TO USE GEOMETRIC PROPERTIES, RELATIONSHIPS, AND MEASUREMENT TO MODEL, DESCRIBE AND ANALYZE PHENOMENA.

Descriptive Statement: Spatial sense is an intuitive feel for shape and space. Geometry and measurement both involve describing the shapes we see all around us in art, nature, and the things we make. Spatial sense, geometric modeling, and measurement can help us to describe and interpret our physical environment and to solve problems.

Geometric Properties. This includes identifying, describing and classifying standard geometric objects, describing and comparing properties of geometric objects, making conjectures concerning them, and using reasoning and proof to verify or refute conjectures and theorems. Also included here are such concepts as symmetry, congruence, and similarity.

Transforming Shapes. Analyzing how various transformations affect geometric objects allows students to enhance their spatial sense. This includes combining shapes to form new ones and decomposing complex shapes into simpler ones. It includes the standard geometric transformations of translation (slide), reflection (flip), rotation (turn), and dilation (scaling). It also includes using tessellations and fractals to create geometric patterns.

Coordinate Geometry. Coordinate geometry provides an important connection between geometry and algebra. It facilitates the visualization of algebraic relationships, as well as an analytical understanding of geometry.

Units of Measurement. Measurement helps describe our world using numbers. An understanding of how we attach numbers to real-world phenomena, familiarity with common measurement units (e.g., inches, liters, and miles per hour), and a practical knowledge of measurement tools and techniques are critical for students' understanding of the world around them.

Measuring Geometric Objects. This area focuses on applying the knowledge and understandings of units of measurement in order to actually perform measurement. While students will eventually apply formulas, it is important that they develop and apply strategies that derive from their understanding of the attributes. In addition to measuring objects directly, students apply indirect measurement skills, using, for example, similar triangles and trigonometry.

Students of all ages should realize that geometry and measurement is all around them. Through study of these areas and their applications, they should come to better understand and appreciate the role of mathematics in their lives.

Building upon knowledge and skills gained in preceding grades, by the end of **Grade 8**, students will:

GEOMETRIC PROPERTIES

PROPERTIES	<p>Understand and apply concepts involving Lines, angles, and planes:</p> <ul style="list-style-type: none"> <input type="checkbox"/> Complementary and supplementary angles <input type="checkbox"/> Vertical angles <input type="checkbox"/> Bisectors and perpendicular bisectors <input type="checkbox"/> Parallel, perpendicular, and intersecting planes <input type="checkbox"/> Intersection of plane with cube, cylinder, cone, and sphere <p>Understand and apply the Pythagorean Theorem to find the missing sides of right triangles</p> <p>Understand and apply properties of polygon</p> <ul style="list-style-type: none"> <input type="checkbox"/> Regular polygons <input type="checkbox"/> Quadrilaterals, including squares, rectangles, parallelograms, trapezoids, rhombi: <input type="checkbox"/> Sum of measures of interior angles of a polygon <input type="checkbox"/> Which polygons can be used alone to generate a tessellation and why <p><input type="checkbox"/> Understand and apply the concept of similarity.</p> <ul style="list-style-type: none"> <input type="checkbox"/> Use proportions to find missing measures <input type="checkbox"/> Prepare scale drawings <input type="checkbox"/> Create Models of 3D objects <input type="checkbox"/> Use logic and reasoning to make and support conjectures about geometric objects.
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TRANSFORMING SHAPES

TRANSFORMING SHAPES	<ul style="list-style-type: none"> <input type="checkbox"/> Understand and apply transformations. <input type="checkbox"/> Find the image, given the pre-image, and vice-versa <input type="checkbox"/> Find the sequence of transformations needed to map one figure onto another <input type="checkbox"/> Find the reflections, rotations, and translations result in images congruent to the pre-image <input type="checkbox"/> Find the dilations (stretching/shrinking) result in images similar to the pre-image <input type="checkbox"/> Use iterative procedures to generate geometric patterns to: <input type="checkbox"/> Explore fractals (e.g., the Koch Snowflake) <input type="checkbox"/> Explore Self-similarity <input type="checkbox"/> Explore patterns in successive stages (e.g., number of triangles in each stage of Sierpinski's Triangle)
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COORDINATE GEOMETRY

COORDINATES	<input type="checkbox"/> Use coordinates in four quadrants to represent geometric concepts. <input type="checkbox"/> Use a coordinate grid to model and quantify transformations (e.g., translate right 4 units).
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UNITS OF MEASUREMENT

UNITS OF MEASUREMENT	<input type="checkbox"/> Solve problems requiring calculations that involve different units of measurement within a measurement system (e.g., 4'3" plus 7'10" equals 12'1"). <input type="checkbox"/> Use approximate equivalents between standard and metric systems to estimate measurements (e.g., 5 kilometers is about 3 miles). <input type="checkbox"/> Recognize that the degree of precision needed in calculations depends on how the results will be used and the instruments used to generate the measurements. <input type="checkbox"/> Select and use appropriate units and tools to measure quantities to the degree of precision needed in a particular problem-solving situation. <input type="checkbox"/> Recognize that all measurements of continuous quantities are approximations. <input type="checkbox"/> Solve problems that involve compound measurement units, such as speed (miles per hour), air pressure (pounds per square inch), and population density (persons per square mile).
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LENGTH	<p style="text-align: center;"><i>Review, Extend, Use and Maintain all Measures In and Through Problem Solving</i></p> <input type="checkbox"/> Fractions of an inch, mile, decimeter, kilometer (1/8, 1/4, 1/2), <input type="checkbox"/> Measure and calculate: Area, length, width using ruler or yardstick, one inch squares, one centimeter squares <input type="checkbox"/> Explore problems using: inches, feet, yards and mile, millimeters, centimeters, meters, kilometers <input type="checkbox"/> Add and subtract customary units of length and width
CAPACITY	<input type="checkbox"/> Solve word problems for: ounce, cup, pint, quart, gallon, milliliter <input type="checkbox"/> Solve problems for milliliter, liter, kiloliter
MASS	<input type="checkbox"/> Measure and calculate: Volume using one inch cubes and one centimeter cubes <input type="checkbox"/> Solve problems using grams, kilograms
WEIGHT	<input type="checkbox"/> Solve problems for: ounce, pounds, ton
VOLUME	<input type="checkbox"/> Solve problems for: cubic inch, cubic centimeter
AREA	<input type="checkbox"/> Solve problems for: square inch, square centimeter
TIME	<input type="checkbox"/> Solve problems involving elapsed time within a twelve hour period <input type="checkbox"/> Measure and calculate: Elapsed time to the nearest five minutes during the same twelve hour period
TEMPERATURE	<input type="checkbox"/> Read and interpret thermometers by both positive and negative degrees OC = 32F

MEASURING GEOMETRIC OBJECTS

MEASURING OBJECTS

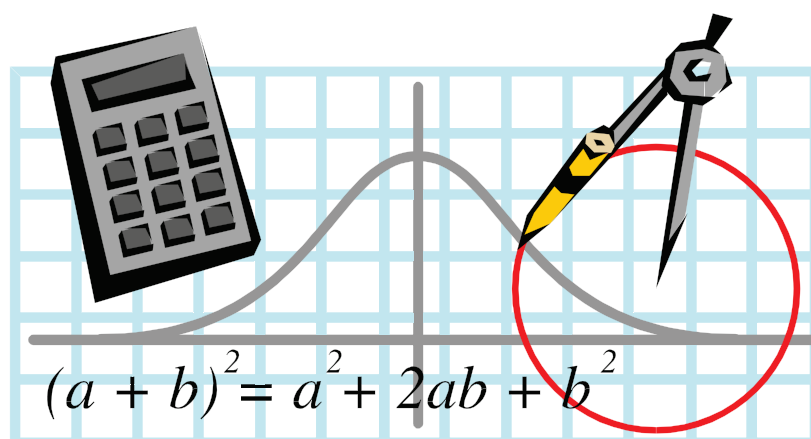
- Develop and apply strategies for finding **perimeter and area**
- Develop and apply strategies for **geometric figures** made by **combining** triangles, rectangles and circles or parts of circles
- Develop and apply strategies for **estimation of area** using grids of various sizes

- Develop and apply strategies for **impact of a dilation** on the perimeter and area of a 2-dimensional figure
- Recognize that the **volume** of a pyramid or cone is one-third of the volume of the prism or cylinder with the same base and height (e.g., use rice to compare volumes of figures with same base and height).

- Develop and apply strategies and formulas for finding the **surface area and volume** of a three-dimensional figure.

- Solve problems with: volume - prism, cone, pyramid
- Solve problems showing surface area - prism (triangular or rectangular base), pyramid (triangular or rectangular base)
- Solve problems showing impact of a dilation on the surface area and volume of a three-dimensional figure

- Use formulas to find the **volume and surface area of a sphere**.



STANDARD 4.3 (PATTERNS AND ALGEBRA) ALL STUDENTS WILL REPRESENT AND ANALYZE RELATIONSHIPS AMONG VARIABLE QUANTITIES AND SOLVE PROBLEMS INVOLVING PATTERNS, FUNCTIONS, AND ALGEBRAIC CONCEPTS AND PROCESSES.

Descriptive Statement: Algebra is a symbolic language used to express mathematical relationships. Students need to understand how quantities are related to one another, and how algebra can be used to concisely express and analyze those relationships. Modern technology provides tools for supplementing the traditional focus on algebraic procedures, such as solving equations, with a more visual perspective, with graphs of equations displayed on a screen. Students can then focus on understanding the relationship between the equation and the graph, and on what the graph represents in a real-life situation.

Patterns. Algebra provides the language through which we communicate the patterns in mathematics. From the earliest age, students should be encouraged to investigate the patterns that they find in numbers, shapes, and expressions, and, by doing so, to make mathematical discoveries. They should have opportunities to analyze, extend, and create a variety of patterns and to use pattern-based thinking to understand and represent mathematical and other real-world phenomena.

Functions and Relationships. The function concept is one of the most fundamental unifying ideas of modern mathematics. Students begin their study of functions in the primary grades, as they observe and study patterns. As students grow and their ability to abstract matures, students form rules, display information in a table or chart, and write equations, which express the relationships they have observed. In high school, they use the more formal language of algebra to describe these relationships.

Modeling. Algebra is used to model real situations and answer questions about them. This use of algebra requires the ability to represent data in tables, pictures, graphs, equations or inequalities, and rules. Modeling ranges from writing simple number sentences to help solve story problems in the primary grades to using functions to describe the relationship between two variables, such as the height of a pitched ball over time. Modeling also includes some of the conceptual building blocks of calculus, such as how quantities change over time and what happens in the long run (limits).

Procedures. Techniques for manipulating algebraic expressions – procedures – remain important, especially for students who may continue their study of mathematics in a calculus program. Utilization of algebraic procedures includes understanding and applying properties of numbers and operations, using symbols and variables appropriately, working with expressions, equations, and inequalities, and solving equations and inequalities.

Algebra is a gatekeeper for the future study of mathematics, science, the social sciences, business, and a host of other areas. In the past, algebra has served as a filter, screening people out of these opportunities. For New Jersey to be part of the global society, it is important that algebra play a major role in a mathematics program that opens the gates for all students.

Building upon knowledge and skills gained in preceding grades, by the end of **Grade 8**, students will:

PATTERNS

PATTERNS	<ul style="list-style-type: none"> <input type="checkbox"/> Create, recognize and explain numerical patterns that include whole numbers, rational numbers, integers, decimal and fractions, <input type="checkbox"/> Recognize, describe arithmetic sequences (i.e., sequences generated by repeated addition of a fixed number, positive or negative) <input type="checkbox"/> Recognize, describe finite and infinite sequences <input type="checkbox"/> Recognize, describe geometric sequences (i.e., sequences generated by repeated multiplication by a fixed positive ratio, greater than 1 or less than 1) <input type="checkbox"/> Recognize, describe descriptions using tables, verbal and symbolic rules, graphs, simple equations or expressions <input type="checkbox"/> Generating sequences by using calculators to repeatedly apply a formula
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FUNCTIONS AND RELATIONSHIPS

Note: Do not assume that the students have a strong background in functions and relationships. Do a pretest of Functions and Relationships, Modeling, Procedures from Grades 5-7 to see if the students have a strong underpinning of ‘pre-algebra’ essential knowledge. You cannot go ahead to do the work of Grade 8 if the students do not have this foundation.

FUNCTIONS AND RELATIONSHIPS	<ul style="list-style-type: none"> <input type="checkbox"/> Graph functions and understand and describe their general behavior that involve: <input type="checkbox"/> Solve equations involving two variables <input type="checkbox"/> Solve problems with rate of change (informal notion of slope.) <input type="checkbox"/> Recognize and describe the difference between linear and exponential growth, using tables, graphs, and equations
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MODELING

MODELING	<ul style="list-style-type: none"> <input type="checkbox"/> Analyze functional relationships to explain how a change in one quantity can result in a change in another, using pictures, graphs, charts, and equations. <input type="checkbox"/> Use patterns, relations, symbolic algebra, and linear functions to model situations. <input type="checkbox"/> Using concrete materials (manipulatives), tables, graphs, verbal rules, algebraic expressions/equations/inequalities <input type="checkbox"/> Growth situations, such as population growth and compound interest, using recursive (e.g., NOW-NEXT) formulas
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PROCEDURES

PROCEDURES	<p>Use graphing techniques on a number line show:</p> <ul style="list-style-type: none"><input type="checkbox"/> Absolute value<input type="checkbox"/> Do arithmetic operations represented by vectors (arrows) (e.g., “-3 + 6” is “left 3, right 6”)<input type="checkbox"/> Solve simple linear equations informally, graphically, and using formal algebraic methods do problems with multi-step, integer coefficients only (although answers may not be integers)<input type="checkbox"/> Solve simple linear inequalities<input type="checkbox"/> Create, evaluate, and simplify algebraic expressions involving variables.<input type="checkbox"/> Show order of operations, including appropriate use of parentheses<input type="checkbox"/> Demonstrate the distributive property<input type="checkbox"/> Demonstrate substitution of a number for a variable<input type="checkbox"/> Show translation of a verbal phrase or sentence into an algebraic expression, equation, or inequality and vice versa<input type="checkbox"/> Use software to show the translation of a verbal phrase or sentence into an algebraic expression, equation, or inequality, and vice versa<input type="checkbox"/> Understand and apply the properties of operations, numbers, equations, and inequalities.<input type="checkbox"/> Show the additive inverse<input type="checkbox"/> Show the multiplicative inverse<input type="checkbox"/> Show the addition and multiplication properties of equality<input type="checkbox"/> Show the addition and multiplication properties of inequalities
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STANDARD 4.4 (DATA ANALYSIS, PROBABILITY, AND DISCRETE MATHEMATICS) ALL STUDENTS WILL DEVELOP AN UNDERSTANDING OF THE CONCEPTS AND TECHNIQUES OF DATA ANALYSIS, PROBABILITY, AND DISCRETE MATHEMATICS, AND WILL USE THEM TO MODEL SITUATIONS, SOLVE PROBLEMS, AND ANALYZE AND DRAW APPROPRIATE INFERENCES FROM DATA.

Descriptive Statement: Data analysis, probability, and discrete mathematics are important interrelated areas of applied mathematics. Each provides students with powerful mathematical perspectives on everyday phenomena and with important examples of how mathematics is used in the modern world. Two important areas of discrete mathematics are addressed in this standard; a third area, iteration and recursion, is addressed in Standard 4.3 (Patterns and Algebra).

Data Analysis (or Statistics). In today's information-based world, students need to be able to read, understand, and interpret data in order to make informed decisions. In the early grades, students should be involved in collecting and organizing data, and in presenting it using tables, charts, and graphs. As they progress, they should gather data using sampling, and should increasingly be expected to analyze and make inferences from data, as well as to analyze data and inferences made by others.

Probability. Students need to understand the fundamental concepts of probability so that they can interpret weather forecasts, avoid unfair games of chance, and make informed decisions about medical treatments whose success rate is provided in terms of percentages. They should regularly be engaged in predicting and determining probabilities, often based on experiments (like flipping a coin 100 times), but eventually based on theoretical discussions of probability that make use of systematic counting strategies. High school students should use probability models and solve problems involving compound events and sampling.

Discrete Mathematics—Systematic Listing and Counting. Development of strategies for listing and counting can progress through all grade levels, with middle and high school students using the strategies to solve problems in probability. Primary students, for example, might find all outfits that can be worn using two coats and three hats; middle school students might systematically list and count the number of routes from one site on a map to another; and high school students might determine the number of three-person delegations that can be selected from their class to visit the mayor.

Discrete Mathematics—Vertex-Edge Graphs and Algorithms. Vertex-edge graphs, consisting of dots (vertices) and lines joining them (edges), can be used to represent and solve problems based on real-world situations. Students should learn to follow and devise lists of instructions, called "algorithms," and use algorithmic thinking to find the best solution to problems like those involving vertex-edge graphs, but also to solve other problems.

These topics provide students with insight into how mathematics is used by decision-makers in our society, and with important tools for modeling a variety of real-world situations. Students will better understand and interpret the vast amounts of quantitative data that they are exposed to daily, and they will be able to judge the validity of data-supported arguments.

Building upon knowledge and skills gained in preceding grades, by the end of **Grade 8**, students will:

DATA ANALYSIS

DATA ANALYSIS	<ul style="list-style-type: none"><input type="checkbox"/> Select and use appropriate representations for sets of data, and measures of central tendency (mean, median, and mode).<input type="checkbox"/> Use the calculator and computer software to find the effect of additional data on measures of central tendency.<input type="checkbox"/> Select and use appropriate representations for type of display most appropriate for given data ...tables, graphs....<input type="checkbox"/> Select and use appropriate representations for box-and-whisker plot, upper quartile, lower quartile <input type="checkbox"/> Select and use appropriate representations for scatter plot<input type="checkbox"/> Use calculators and computer software to find the median and mean (weighted average) using frequency data.<input type="checkbox"/> Make inferences and formulate and evaluate arguments based on displays and analysis of data.<input type="checkbox"/> Estimate lines of best fit and use them to interpolate within the range of the data.<input type="checkbox"/> Use surveys and sampling techniques to generate data and draw conclusions about large groups.
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PROBABILITY

PROBABILITY	<ul style="list-style-type: none"><input type="checkbox"/> Interpret probabilities as ratios, percents, and decimals.<input type="checkbox"/> Determine probabilities of simple and compound events.<input type="checkbox"/> Explore the probabilities of conditional events (e.g., if there are seven marbles in a bag, three red and four green, what is the probability that two marbles picked from the bag, without replacement, are both red).<input type="checkbox"/> Model situations involving probability with simulations (using spinners, dice, calculators and computers) and theoretical models. <input type="checkbox"/> Find Frequency, relative frequency of data<input type="checkbox"/> Estimate probabilities and make predictions based on experimental and theoretical probabilities.<input type="checkbox"/> Play and analyze probability-based games, and discuss the concepts of fairness and expected value.
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Discrete Mathematics—Systematic Listing and Counting

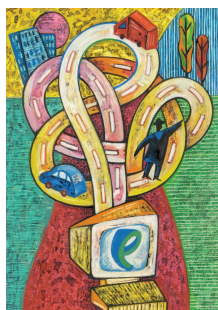
LISTING AND COUNTING

- Apply the **multiplication principle** of counting.
- Explore **permutations**: Ordered situations with replacement (e.g., number of possible license plates) vs. ordered situations without replacement (e.g., number of possible slates of 3 class officers from a 23 student class)
- Understand **factorial notation**
- Understand the concept of **combinations** (e.g., number of possible delegations of 3 out of 23 students)
- Explore counting problems involving Venn diagrams with three attributes (e.g., there are 15, 20, and 25 students respectively in the chess club, the debating team, and the engineering society; how many different students belong to the three clubs if there are 6 students in chess and debating, 7 students in chess and engineering, 8 students in debating and engineering, and 2 students in all three?).
- Apply techniques of **systematic listing, counting, and reasoning** in a variety of different contexts.

Discrete Mathematics—Vertex-Edge Graphs and Algorithms

VERTEX-EDGE GRAPHS AND ALGORITHMS

- Use vertex-edge graphs and algorithmic thinking to represent and find solutions to practical problems as you search for ways of:
 - Finding the shortest network connecting specified sites
 - Finding a minimal route that includes every street (e.g., for trash pick-up)
 - Finding the shortest route on a map from one site to another
 - Finding the shortest circuit on a map that makes a tour of specified sites
- Understand the limitations of computers (e.g., the number of routes for a delivery truck visiting n sites is $n!$, so finding the shortest circuit by examining all circuits would overwhelm the capacity of any computer, now or in the future, even if n is less than 100)



STANDARD 4.5 (MATHEMATICAL PROCESSES) ALL STUDENTS WILL USE MATHEMATICAL PROCESSES OF PROBLEM SOLVING, COMMUNICATION, CONNECTIONS, REASONING, REPRESENTATIONS, AND TECHNOLOGY TO SOLVE PROBLEMS AND COMMUNICATE MATHEMATICAL IDEAS.

Descriptive Statement: The mathematical processes described here highlight ways of acquiring and using the content knowledge and skills delineated in the first four mathematics standards.

Problem Solving. Problem posing and problem solving involve examining situations that arise in mathematics and other disciplines and in common experiences, describing these situations mathematically, formulating appropriate mathematical questions, and using a variety of strategies to find solutions. Through problem solving, students experience the power and usefulness of mathematics. Problem solving is interwoven throughout the grades to provide a context for learning and applying mathematical ideas.

Communication. Communication of mathematical ideas involves students' sharing their mathematical understandings in oral and written form with their classmates, teachers, and parents. Such communication helps students clarify and solidify their understanding of mathematics and develop confidence in themselves as mathematics learners. It also enables teachers to better monitor student progress.

Connections. Making connections involves seeing relationships between different topics, and drawing on those relationships in future study. This applies within mathematics, so that students can translate readily between fractions and decimals, or between algebra and geometry; to other content areas, so that students understand how mathematics is used in the sciences, the social sciences, and the arts; and to the everyday world, so that students can connect school mathematics to daily life.

Reasoning. Mathematical reasoning is the critical skill that enables a student to make use of all other mathematical skills. With the development of mathematical reasoning, students recognize that mathematics makes sense and can be understood. They learn how to evaluate situations, select problem-solving strategies, draw logical conclusions, develop and describe solutions, and recognize how those solutions can be applied.

Representations. Representations refer to the use of physical objects, drawings, charts, graphs, and symbols to represent mathematical concepts and problem situations. By using various representations, students will be better able to communicate their thinking and solve problems. Using multiple representations will enrich the problem solver with alternative perspectives on the problem. Historically, people have developed and successfully used manipulatives (concrete representations such as fingers, base ten blocks, Geoboards, and algebra tiles) and other representations (such as coordinate systems) to help them understand and develop mathematics.

Technology. Calculators and computers need to be used along with other mathematical tools by students in both instructional and assessment activities. These tools should be used, not to replace mental math and paper-and-pencil computational skills, but to enhance understanding of mathematics and the power to use mathematics. Students should explore both new and familiar concepts with calculators and computers and should also become proficient in using technology as it is used by adults (e.g., for assistance in solving real-world problems).

At each grade level, with respect to content appropriate for that grade level, students will:

A. PROBLEM SOLVING

- Learn mathematics through problem solving, inquiry, and discovery.
- Solve problems that arise in mathematics and in other contexts (cf. workplace readiness standard 8.3).
- Solve open-ended problems
- Solve non-routine problems
- Solve problems with multiple solutions
- Solve problems that can be solved in several ways
- Select and apply a variety of appropriate problem-solving strategies (e.g., “try a simpler problem” or “make a diagram”) to solve problems.
- Pose problems of various types and levels of difficulty.
- Monitor their progress and reflect on the process of their problem solving activity.

B. Communication

- Use communication to organize and clarify their mathematical thinking.
- Read and write often in math class
- Use Discussion, listening, and questioning techniques to:
 - Communicate mathematical thinking coherently and clearly to peers, teachers, and others, both orally and in writing.
 - Analyze and evaluate the mathematical thinking and strategies of others.
 - Use the language of mathematics to express mathematical ideas precisely.

C. Connections

- Recognize recurring themes across mathematical domains (e.g., patterns in number, algebra, and geometry).
- Use connections among mathematical ideas to explain concepts (e.g., two linear equations have a unique solution because the lines they represent intersect at a single point).
- Recognize that mathematics is used in a variety of contexts outside of mathematics.
- Apply mathematics in practical situations and in other disciplines.
- Trace the development of mathematical concepts over time and across cultures (cf. world languages and social studies standards).
- Understand how mathematical ideas interconnect and build on one another to produce a coherent whole.

D. Reasoning

- Recognize that mathematical facts, procedures, and claims must be justified.
- Use reasoning to support their mathematical conclusions and problem solutions.
- Select and use various types of reasoning and methods of proof.
- Rely on reasoning, rather than answer keys, teachers, or peers, to check the correctness of their problem solutions.
- Make and investigate mathematical conjectures.
- Counter examples as a means of disproving conjectures
- Verifying conjectures using informal reasoning or proofs.
 - Evaluate examples of mathematical reasoning and determine whether they are valid.

E. REPRESENTATIONS

- Create and use representations to organize, record, and communicate mathematical ideas
 - Concrete representations (e.g., base-ten blocks or algebra tiles)
 - Pictorial representations (e.g., diagrams, charts, or tables)
 - Symbolic representations (e.g., a formula)
 - Graphical representations (e.g., a line graph)
- Select, apply, and translate among mathematical representations to solve problems.
- Use representations to model and interpret physical, social, and mathematical phenomena.

F. TECHNOLOGY

- Use technology to gather, analyze, and communicate mathematical information.
- Use computer spreadsheets, software, and graphing utilities to organize and display quantitative information (cf. workplace readiness standard 8.4-D).
- Use graphing calculators and computer software to investigate properties of functions and their graphs.
- Use calculators as problem-solving tools (e.g., to explore patterns, to validate solutions).
- Use computer software to make and verify conjectures about geometric objects.
- Use computer-based laboratory technology for mathematical applications in the sciences (cf. science standards).