Mathematics Areas of Focus: Grade 5

Mission: *Through mathematics, students communicate, make connections, reason, and represent the world quantitatively in order to pose and solve problems.*

Standard 4.1 Number and Numerical Operations

All students will develop number sense and will perform standard numerical operations and estimations on all types of numbers in a variety of ways.

Big Idea: Numeric reasoning involves fluency and facility with numbers.

4.1.5 A. Number Sense

Descriptive Statement: *Number sense is an intuitive feel for numbers and a common sense approach to using them. It is a comfort with what numbers represent that comes from investigating their characteristics and using them in diverse situations. It involves an understanding of how different types of numbers, such as fractions and decimals, are related to each other, and how each can best be used to describe a particular situation. It subsumes the more traditional category of school mathematics curriculum called numeration and thus includes the important concepts of place value, number base, magnitude, and approximation and estimation.*

Essential Questions	Enduring Understandings
▪ How do mathematical ideas interconnect and build on one another to produce a coherent whole? (4.5C1; 4.5C6)** ▪ How can we compare and contrast numbers? (4.5A4)** ▪ How can counting, measuring, or labeling help to make sense of the world around us?	▪ One representation may sometimes be more helpful than another; and, used together, multiple representations give a fuller understanding of a problem. ▪ A quantity can be represented numerically in various ways. Problem solving depends upon choosing wise ways. ▪ Numeric fluency includes both the understanding of and the ability to appropriately use numbers.

Areas of Focus	Comments and Examples
1. **Use real-life experiences, physical materials, and technology to construct meanings for numbers (unless otherwise noted, all indicators for grade 5 pertain to these sets of numbers as well): All fractions as part of a whole, as subset of a set, as a location on a number line, and as divisions of whole numbers; All decimals.**	It is important to note that the sets of numbers specified in this CPI also apply to the other grade 5 mathematics CPIs, including for example 4.1.5A3 and 4.1.5A6 below. ***Sample Short Constructed Response (SCR) Item:*** *Four friends have three brownies left over from a party. They would like to split them equally. How much should each of them receive? (Answer: 75% or .75 or 3/4 of a brownie)*
2. **Recognize the decimal nature of United States currency and compute with money.**	**Assessment Focus:** • The emphasis in statewide assessment is on the computation. ***Sample Multiple Choice (MC) Item:*** *Debbie has a $5 bill. She wants to purchase a notebook for 75¢ and a pen for 50¢. How much money will Debbie have left after purchasing the notebook and the pen?* *a. $1.25 b. $2.75 * c. $3.75 d. $4.25* ***Sample Short Constructed Response (SCR) Item:*** *Juliette has a $5 bill. She wants to purchase a notebook for 75¢ and a pen for 50¢. How much money will Juliette have left after purchasing the notebook and the pen? (Answer: $3.75)*
3. **Demonstrate a sense of the relative magnitudes of numbers.**	**Instructional/Assessment Focus:** • Refers not only to whole numbers, but also to fractions and decimals, as specified in 4.1.5A1. ***Sample MC Item:*** *If these fractions were graphed on the number line, which of them would be closest to zero?* *a. $\frac{3}{5}$ b. $\frac{1}{4}$ c. $\frac{3}{20}$ * d. $\frac{1}{10}$*
4. **Use whole numbers, fractions, and decimals to represent equivalent forms of the same number.**	***Sample MC Item:*** *Which of the following is equivalent to 3/4?* *a. .25 b. $\frac{4}{3}$ c. .85 * d. $\frac{9}{12}$*
5. **Develop and apply number theory concepts in problem solving situations: Primes, factors, multiples.**	**Assessment Focus:** • The emphasis in statewide assessment is on application. ***Sample MC Item:*** *How many numbers between 20 and 50 have no remainder when divided by 6?* *a. 3 b. 4 * c. 5 d. 6*

Focal points at this grade level are BOLDED
*Correct answer to a multiple-choice item
**Process Standard 4.5 imbedded in content

6. **Compare and order numbers.**	**Instructional/Assessment Focus:** • Refers not only to whole numbers, but also to fractions and decimals, as specified in 4.1.5A1. ***Sample SCR Item:*** *State a number that is between 1/3 and 0.36. Acceptable answers would include various representations of Real Numbers between 1/3 and .36 (e.g., 0.34, 0.334, 0.35, 7/20, etc.)* ***Sample Extended Constructed Response (ECR) Item:*** *On the number line in your answer folder, plot points for the following numbers.* *4/5, 0.6* • *Label each point.* • *Name two different rational numbers that are greater than 0.6 and less than 4/5. (Write one of your numbers in fractional form and write the other number in decimal form.)* • *Explain how you know that each of your numbers is greater than 0.6 and less than 4/5.*

4.1.5 B. Numerical Operations

Descriptive Statement: *Numerical Operations are an essential part of the mathematics curriculum, especially in the elementary grades. Students must be able to select and apply various computational methods, including mental math, pencil-and-paper techniques, and the use of calculators. Students must understand how to add, subtract, multiply, and divide whole numbers, fractions, decimals, and other kinds of numbers. With the availability of calculators that perform these operations quickly and accurately, the instructional emphasis now is on understanding the meanings and uses of these operations, and on estimation and mental skills, rather than solely on the development of paper-and-pencil proficiency.*

Essential Questions	Enduring Understandings
▪ What makes a computational strategy both effective and efficient? (4.5D1)**	▪ Computational fluency includes understanding not only the meaning, but also the appropriate use of numerical operations.
▪ How do operations affect numbers?	▪ The magnitude of numbers affects the outcome of operations on them.
▪ How do mathematical representations reflect the needs of society across cultures? (An essential question with broad applicability across multiple standards) (4.5C5)**	▪ In many cases, there are multiple algorithms for finding a mathematical solution, and those algorithms are frequently associated with different cultures.

Areas of Focus	Comments and Examples
1. Recognize the appropriate use of each arithmetic operation in problem situations.	**Instructional/Assessment Focus:** • The intent is that students not only recognize the appropriate use of arithmetic operations in the work of others, but that they also be able to appropriately use those operations themselves.
2. Construct, use, and explain procedures for performing addition and subtraction with fractions and decimals with: Pencil-and-paper; Mental math; Calculator.	"Construct" here means "develop" an algorithm or process. ***Sample SCR Item:*** *Paula's tractor holds 3 liters of gasoline. Tom's tractor holds 2.4 liters. How much more does one tractor hold than the other? (Answer: 0.6 liters)* ***Sample ECR Item:*** *Joe had a pizza party. He ordered 8 pizzas, each cut into 8 slices. When his friends went home, he had ¼ of a pepperoni pizza, 5/8 of a mushroom pizza, ½ of a cheese pizza, and 1/8 of a veggie pizza left over. How much pizza was left over in all?* • *Show one way to get the answer to this problem. Explain your method.* • *Show another way to get the answer to this problem. Explain your method.* ***Sample SCR Item:*** *A fifth-grade class will perform an act for the spring talent show. Two-thirds of the class of 24 students want to perform a skit. The rest of the students in the class want to sing a song. The teacher decided that 3/4 of the students must agree on an act before the decision will be final. How many more students would have to choose a skit before ¾ of the students agree on it? (Answer: 2 students)*
3. Use an efficient and accurate pencil-and-paper procedure for division of a 3-digit number by a 2-digit number.	***Sample SCR Item:*** *A gallon contains 128 ounces. Paul wants to divide three gallons of apple cider equally among the two dozen friends at his party. How much apple cider will each friend receive? (Answer: 16 oz.)*
4. Select pencil-and-paper, mental math, or a calculator as the appropriate computational method in a given situation depending on the context and numbers.	Assessment of this CPI is generally within the context of one or more of the other content CPIs.

5. Check the reasonableness of results of computations.	**Instructional/Assessment Focus:** **Includes** • Identifying unreasonable answers obtained using a calculator; • Using inverse operations to check solutions; • Reasoning (4.5D2) and communication (4.5B2)**; • Solving problems (4.5A2)** involving this recognition; and • Application to all fractions and decimals, as specified in 4.1.5A1. ***Sample ECR Item:*** *The fifth grade at Park Middle School is taking a field trip using buses that hold 36 passengers each.. There are three classes of 25 students each, and 5 adults (teachers or parents) will accompany each of the three classes. The Principal wants to order 2 ½ buses; the Superintendent wants to order 2 buses; and the fifth-grade teachers want to order 3 buses. Which suggestion is most reasonable and why? Explain your reasoning.*
6. Understand and use the various relationships among operations and properties of operations.	"Use" here means "apply." The "properties of operations" referred to include those specifically listed in 4.3.2D1, 4.3.3D1, or 4.3.4D1 (commutative properties, identity elements, associative properties, and multiplication or division by zero). **Assessment Focus:** • The emphasis in statewide assessment is on the "use" or "apply," rather than on the "understand."

4.1.5 C. Estimation

Descriptive Statement: *Estimation is a process that is used constantly by mathematically capable adults, and one that can be easily mastered by children. It involves an educated guess about a quantity or an intelligent prediction of the outcome of a computation. The growing use of calculators makes it more important than ever that students know when a computed answer is reasonable; the best way to make that determination is through the use of strong estimation skills. Equally important is an awareness of the many situations in which an approximate answer is as good as, or even preferable to, an exact one. Students can learn to make these judgments and use mathematics more powerfully as a result.*

Essential Questions	Enduring Understandings
▪ How can we decide when to use an exact answer and when to use an estimate?	▪ Context is critical when using estimation.

Areas of Focus	Comments and Examples
1. Use a variety of estimation strategies for both number and computation.	"Number" here refers to "quantities." Assessment of this CPI is generally within the context of one or more of the other content CPIs.
2. Recognize when an estimate is appropriate, and understand the usefulness of an estimate as distinct from an exact answer.	"Understand" implies "explain," consistent with 4.5B1 and 4.5B2. This is an area of focus in grade 4 and may be assessed at a higher level of understanding in grade 5.
3. **Determine the reasonableness of an answer by estimating the result of operations.**	
4. **Determine whether a given estimate is an overestimate or an underestimate.**	

Standard 4.2 Geometry and Measurement

All students will develop spatial sense and the ability to use geometric properties, relationships, and measurement to model, describe and analyze phenomena.

Big Idea *Geometry***:** Spatial sense and geometric relationships are a means to solve problems and make sense of a variety of phenomena.

Big Idea *Measurement***:** Measurement is a tool to quantify a variety of phenomena.

4.2.5 A. Geometric Properties

Descriptive Statement: *This includes identifying, describing and classifying standard geometric object, describing and comparing properties of geometric objects, making conjectures concerning them, and using reasoning and proof to verify or refute conjectures and theorems. Also included here are such concepts as symmetry, congruence, and similarity.*

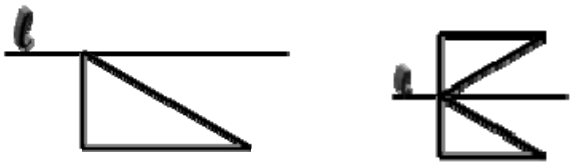
Essential Questions	Enduring Understandings
▪ How can spatial relationships be described by careful use of geometric language?	▪ Geometric properties can be used to construct geometric figures. (4.5D1; 4.5D2; 4.5E3)**
▪ How do geometric relationships help in solving problems and/or make sense of phenomena?	▪ Geometric relationships provide a means to make sense of a variety of phenomena.

Areas of Focus	Comments and Examples
1. Understand and apply concepts involving lines and angles: Notation for line, ray, angle, line segment; Properties of parallel, perpendicular, and intersecting lines; Sum of the measures of the interior angles of a triangle is 180°.	"Understand and apply" here means "define, recognize, and apply."
2. Identify, describe, compare, and classify polygons: Triangles by angles and sides; Quadrilaterals, including squares, rectangles, parallelograms, trapezoids, rhombi; Polygons by number of sides; Equilateral, equiangular, regular; All points equidistant from a given point form a circle.	
3. Identify similar figures.	
4. Understand and apply the concepts of congruence and symmetry (line and rotational).	

4.2.5 B. Transforming Shapes

Descriptive Statement: *This includes identifying, describing and classifying standard geometric object, describing and comparing properties of geometric objects, making conjectures concerning them, and using reasoning and proof to verify or refute conjectures and theorems. Also included here are such concepts as symmetry, congruence, and similarity.*

Essential Questions	Enduring Understandings
▪ What situations can be analyzed using transformations and symmetries? (4.5E1; 4.5E2; 4.5E3)**	▪ Shape and area can be conserved during mathematical transformations.

Areas of Focus	Comments and Examples
1. Use a translation, a reflection, or a rotation to map one figure onto another congruent figure.	***Sample SCR Item:*** *Show the result of reflecting the triangle across line l.* *Answer:* 
2. Recognize, identify, and describe geometric relationships and properties as they exist in nature, art, and other real-world settings.	***Sample SCR Item:*** *Draw the line of symmetry in the figure below.* *Answer:* 

4.2.5 C. Coordinate Geometry

Descriptive Statement: *Coordinate geometry provides an important connection between geometry and algebra. It facilitates the visualization of algebraic relationships, as well as an analytical understanding of geometry.*

Essential Questions	Enduring Understandings
▪ How can geometric/algebraic relationships best be represented and verified? (4.5C2; 4.5D2; 4.5E1; 4.5E2; 4.5F5)**	▪ Reasoning and/or proof can be used to verify or refute conjectures or theorems in geometry (4.5D1; 4.5D3; 4.5D4; 4.5D5; 4.5F5)** ▪ Coordinate geometry can be used to represent and verify geometric/algebraic relationships.

Areas of Focus	Comments and Examples
1. Create geometric shapes with specified properties in the first quadrant on a coordinate grid.	***Sample SCR Item:*** *Three vertices of a parallelogram are at the points (0, 0), (2, 4), and (6, 0). What are the coordinates of the fourth vertex?* *(Answer: (8,4) or (-4,4) or (4, -4). Although not expected to find either of the answers out of the first quadrant, a student would not be penalized for finding such a vertex.)*

4.2.5 D. Units Of Measurement

Descriptive Statement: *Measurement helps describe our world using numbers. An understanding of how we attach numbers to real-world phenomena, familiarity with common measurement units (e.g., inches, liters, and miles per hour), and a practical knowledge of measurement tools and techniques are critical for students' understanding of the world around them.*

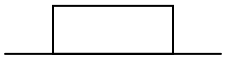
Essential Questions	Enduring Understandings
▪ How can measurements be used to solve problems? (4.5A6)**	▪ Everyday objects have a variety of attributes, each of which can be measured in many ways. ▪ What we measure affects how we measure it. (4.5A4; 4.5A6)** ▪ Measurements can be used to describe, compare, and make sense of phenomena.

Areas of Focus	Comments and Examples
1. Select and use appropriate units to measure angles and area.	***Sample MC Item:*** *What units would most likely be used to measure the area of a classroom floor?* *a. Square inches * b. Square feet c. Cubic feet d. Cubic yards*
2. Convert measurement units within a system (e.g., 3 feet = ___ inches).	***Sample ECR Item:*** *Two students measured the same book shelf. Debbie said the measurement is 3. Tim said the measurement is 36. How can both students be correct? Explain your reasoning.*
3. Know approximate equivalents between the standard and metric systems (e.g., one kilometer is approximately 6/10 of a mile).	"Know approximate equivalents" means that students should be able to recognize or produce approximate equivalents. ***Sample ECR Item:*** *Carol measured her height to be 1.5. How can this be possible? Explain your reasoning.*
4. Use measurements and estimates to describe and compare phenomena.	This CPI will infrequently be measured independently, but will provide a context for measuring other CPIs.

4.2.5 E. Measuring Geometric Objects

Descriptive Statement: *This area focuses on applying the knowledge and understandings of units of measurement in order to actually perform measurement. While students will eventually apply formulas, it is important they develop and apply strategies that derive from their understanding of the attributes. In addition to measuring objects directly, students apply indirect measurement skills, using, for example, similar triangles and trigonometry.*

Essential Questions	Enduring Understandings
▪ How can measurements be used to solve problems? (4.5A6)**	▪ Everyday objects have a variety of attributes, each of which can be measured in many ways. ▪ What we measure affects how we measure it. (4.5A4; 4.5A6)** ▪ Measurements can be used to describe, compare, and make sense of phenomena.

Areas of Focus	Comments and Examples
1. Use a protractor to measure angles.	
2. Develop and apply strategies and formulas for finding perimeter and area: Square; Rectangle.	***Sample ECR Item:*** *Linda wants to fence in a rectangular yard for her dogs to run around and get exercise. She only has 72 feet of fencing, so she decides to use the side of the barn as one side of the fenced-in yard.* *?*  *Barn Wall* • *Sketch a graph showing the relationship between the width of the yard and the area. Remember that the lengths of the three sides of the yard have to add up to 72 feet. The fourth side of the yard is formed by the side of the barn.* • *Use the graph to determine which width would result in the greatest area for the dogs to run in. Explain your reasoning.*
3. Recognize that rectangles with the same perimeter do not necessarily have the same area and vice versa.	Assessment of this CPI is generally within the context of CPI 4.2.5E2. **Assessment Focus:** • Students are expected to solve problems (4.5A2)** involving this recognition.
4. Develop informal ways of approximating the measures of familiar objects (e.g., use a grid to approximate the area of the bottom of one's foot).	

Standard 4.3 Patterns and Algebra

All students will represent and analyze relationships among variable quantities and solve problems involving patterns, functions, and algebraic concepts and processes.

Big Idea: Algebra provides language through which we communicate the patterns in mathematics.

4.3.5 A. Patterns

Descriptive Statement: *Algebra provides the language through which we communicate the patterns in mathematics. From the earliest age, students should be encouraged to investigate the patterns that they find in numbers, shapes, and expressions, and by doing so, to make mathematical discoveries. They should have opportunities to analyze, extend, and create a variety of patterns and to use pattern-based thinking to understand and represent mathematical and other real-world phenomena.*

Essential Questions	Enduring Understandings
▪ How can change be best represented mathematically? (4.5C1; 4.5F1; 4.5F2; 4.5F3; 4.5F4)** ▪ How can patterns, relations, and functions be used as tools to best describe and help explain real-life situations? (4.5C1)**	▪ The symbolic language of algebra is used to communicate and generalize the patterns in mathematics. ▪ Algebraic representation can be used to generalize patterns and relationships.

Areas of Focus	Comments and Examples
1. Recognize, describe, extend, and create patterns involving whole numbers: Descriptions using tables, verbal rules, simple equations, and graphs.	***Sample MC Item:*** *Last year, the cafeteria at Kyle's school recycled 100 pounds of the trash that was collected. This year was the second year of recycling, and the cafeteria recycled twice as much. If the amount of trash the cafeteria recycles doubles each year, how much will be recycled in the fourth year?* *a. 1600 pounds * b. 800 pounds c. 600 pounds d. 400 pounds*

4.3.5 B. Functions & Relationships

Descriptive Statement: *The function concept is one of the most fundamental unifying ideas of modern mathematics. Student begin their study of functions in the primary grades, as they observe and study patterns. As students grow and their ability to abstract matures, students form rules, display information in a table or chart, and write equations which express the relationships they have observed. In high school, they use the more formal language of algebra to describe these relationships.*

Essential Questions	Enduring Understandings
▪ How are patterns of change related to the behavior of functions? (4.5F1; 4.5F2; 4.5F3; 4.5F4)**	▪ Patterns and relationships can be represented graphically, numerically, symbolically, or verbally. (4.5E1)**

Areas of Focus	Comments and Examples
1. Describe arithmetic operations as functions, including combining operations and reversing them.	
2. Graph points satisfying a function from T-charts, from verbal rules, and from simple equations.	

4.3.5 C. Modeling

Descriptive Statement: *Algebra is used to model real situations and answer questions about them. This use of algebra requires the ability to represent data in tables, pictures, graphs, equations or inequalities, and rules. Modeling ranges from writing simple number sentences to help solve story problems in the primary grades to using functions to describe the relationship between two variables, such as the height of a pitched ball over time. Modeling also includes some of the conceptual building blocks of calculus, such as how quantities change over time and what happens in the long run (limits).*

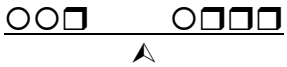
Essential Questions	Enduring Understandings
▪ How are mathematical models used to describe physical relationships? (4.5E2)** ▪ How are physical models used to clarify mathematical relationships? (4.5E3)**	▪ Mathematical models can be used to describe and quantify physical relationships. (4.5E2)** ▪ Physical models can be used to clarify mathematical relationships. (4.5E3)**

Areas of Focus	Comments and Examples
1. Use number sentences to model situations: Using variables to represent unknown quantities; Using concrete materials, tables, graphs, verbal rules, algebraic expressions/equations	
2. Draw freehand sketches of graphs that model real phenomena and use such graphs to predict and interpret events: Changes over time; Rates of change (e.g., when is a plant growing slowly/rapidly, when is temperature dropping most rapidly/slowly).	**Assessment Focus:** • Students are asked to draw a graphical representation of a story.

4.3.5 D. Procedures

Descriptive Statement: *Techniques for manipulating algebraic expressions - procedures - remain important, especially for students who may continue their study of mathematics in a calculus program. Utilization of algebraic procedures includes understanding and applying properties of numbers and operations, using symbols and variables appropriately, working with expressions, equations, and inequalities, and solving equations and inequalities.*

Essential Questions	Enduring Understandings
▪ What makes an algebraic algorithm both effective and efficient? (4.5D1)**	▪ Algebraic and numeric procedures are interconnected and build on one another to produce a coherent whole. ▪ Reasoning and/or proof can be used to verify or refute conjectures or theorems in algebra. (4.5D1; 4.5D3; 4.5D4; 4.5D5)**

Areas of Focus	Comments and Examples
1. Solve simple linear equations with manipulatives and informally: Whole-number coefficients only, answers also whole numbers; Variables on one side of equation.	***Sample SCR Item:*** *What value for A would make the following number sentence true?* *16 - A = 2* *(Answer: 14)* ***Sample MC Item:*** *The scale below shows spheres and cubes balanced. How many cubes balance a sphere?*  *a. 1 b. 1 1/2 * c. 2 d. 3*

Standard 4.4 Data Analysis, Probability, and Discrete Mathematics

All students will develop an understanding of the concepts and techniques of data analysis, probability, and discrete mathematics, and will use them to model situations, solve problems, and analyze and draw appropriate inferences from data.

Big Idea *Data Analysis***:** Reading, understanding, interpreting, and communicating data are critical in modeling a variety of real-world situations, drawing appropriate inferences, making informed decisions, and justifying those decisions.

Big Idea *Probability***:** Probability quantifies the likelihood that something will happen and enables us to make predictions and informed decisions.

Big Idea *Discrete Mathematics***:** Discrete mathematics consists of tools and strategies for representing, organizing, and interpreting non-continuous data.

4.4.5 A. Data Analysis

Descriptive Statement: *In today's information-based world, students need to be able to read, understand, and interpret data in order to make informed decisions. In the early grades, students should be involved in collecting and organizing data, and in presenting it using tables, charts, and graphs. As they progress, they should gather data using sampling, and should increasingly be expected to analyze and make inferences from data, as well as to analyze data and inferences made by others.*

Essential Questions	Enduring Understandings
▪ How can the collection, organization, interpretation, and display of data be used to answer questions? (4.5A4; 4.5A6; 4.5E1; 4.5E2; 4.5F1; 4.5F6)**	▪ The message conveyed by the data depends on how the data is collected, represented, and summarized. (4.5A6; 4.5D6; 4.5E1; 4.5E2; 4.5E3)** ▪ The results of a statistical investigation can be used to support or refute an argument. (4.5D1; 4.5D3; 4.5D5; 4.5E2; 4.5E3; 4.5F6)**

Areas of Focus	Comments and Examples
1. Collect, generate, organize, and display data: Data generated from surveys.	Assessment of this CPI is generally within the context of CPI 4.4.5A2.
2. Read, interpret, select, construct, analyze, generate questions about, and draw inferences from displays of data: Bar graph, line graph, circle graph, table; Range, median, and mean.	***Sample SCR Item:*** *On five tests of 100 points each, José has an average of exactly 90. What is the lowest score he could have made on any of the five tests?*
3. Respond to questions about data and generate their own questions and hypotheses.	

4.4.5 B. Probability

Descriptive Statement: *Students need to understand the fundamental concepts of probability so that they can interpret weather forecasts, avoid unfair games of chance, and make informed decisions about medical treatments whose success rate is provided in terms of percentages. They should regularly be engaged in predicting and determining probabilities, often based on experiments (like flipping a coin 100 times), but eventually based on theoretical discussions of probability that make use of systematic counting strategies. High school students should use probability models and solve problems involving compound events and sampling.*

Essential Questions	Enduring Understandings
▪ How can experimental and theoretical probabilities be used to make predictions or draw conclusions? (4.5D5; 4.5D6)**	▪ Experimental results tend to approach theoretical probabilities after a large number of trials.

Areas of Focus	Comments and Examples
1. Determine probabilities of events: Event, probability of an event; Probability of certain event is 1 and of impossible event is 0.	***Sample SCR Item:*** *Mike has a number cube with the letter "M" on all six faces. What is the probability of his rolling an "M" on his next roll?*
2. Determine probability using intuitive, experimental, and theoretical methods (e.g., using model of picking items of different colors from a bag); Given numbers of various types of items in a bag, what is the probability that an item of one type will be picked; Given data obtained experimentally, what is the likely distribution of items in the bag.	***Sample SCR Item:*** *If there are seven marbles in a bag, three red and four green, what is the probability that a marble picked from the bag will be red? (Answer: 3/7 or 3 out of 7)* ***Sample MC Item:*** *Cynthia has a bag of 10 marbles that contains 4 red marbles and 6 blue marbles. If Cynthia reached into the bag without looking and picked one marble, what is the probability that she would pick a blue marble?* *a. 1/10 b. 2/5 * c. 3/5 d. 1*
3. Model situations involving probability using simulations (with spinners, dice) and theoretical models.	**Instructional Focus:** • This CPI is largely an instructional CPI. Assessment of this CPI is generally within the context of one or more of the other content CPIs.

4.4.5 C. Discrete Mathematics - Systematic Listing And Counting

Descriptive Statement: *Development of strategies for listing and counting can progress through all grade levels, with middle and high school students using the strategies to solve problems in probability. Primary students, for example, might find all outfits that can be worn using two coats and three hats; middle school students might systematically list and count the number of routes from one site on a map to another; and high school students might determine the number of three-person delegations that can be selected from their class to visit the mayor.*

Essential Questions	Enduring Understandings
▪ How can attributes be used to classify data/objects? ▪ What is the best way to solve this? What counting strategy works best here?	▪ Grouping by attributes (classification) can be used to answer mathematical questions. (4.5E1; 4.5E3)** ▪ Algorithms can effectively and efficiently be used to quantify and interpret discrete information.

Areas of Focus	Comments and Examples
1. Solve counting problems and justify that all possibilities have been enumerated without duplication: Organized lists, charts, tree diagrams, tables.	***Sample ECR Item:*** *Out of six students who have expressed interest in student government, represent all possibilities for a slate of three officers, using a list, a chart, or a tree diagram.* ***Sample MC Item:*** *Four fifth-graders are scheduled to have their picture taken as a group. If they are going to stand side-by-side, in how many ways can they be arranged?* *a. 8 b. 12 c. 16 * d. 24*
2. Explore the multiplication principle of counting in simple situations by representing all possibilities in an organized way (e.g., you can make 3 x 4 = 12 outfits using 3 shirts and 4 skirts).	**Instructional Focus:** • This Content should be introduced at this grade level, but mastery of the content is not assessed in statewide assessment at this grade level.

4.4.5 D. Discrete Mathematics - Vertex-Edge Graphs And Algorithms

Descriptive Statement: *Vertex-edge graphs, consisting of dots (vertices) and lines joining them (edges), can be used to represent and solve problems based on real-world situations. Students should learn to follow and devise lists of instructions, called "algorithms," and use algorithmic thinking to find the best solution to problems like those involving vertex-edge graphs, but also to solve other problems.*

Essential Questions	Enduring Understandings
▪ How can visual tools such as networks (vertex-edge graphs) be used to answer questions? (4.5E1; 4.5E3)** ▪ How can algorithmic thinking be used to solve problems?	▪ Optimization is finding the best solution within given constraints. ▪ Algorithms can effectively and efficiently be used to quantify and interpret discrete information.

Areas of Focus	Comments and Examples
1. Devise strategies for winning simple games (e.g., start with two piles of objects, each of two players in turn removes any number of objects from a single pile, and the person to take the last group of objects wins) and express those strategies as sets of directions.	***Sample MC Item:*** *Joe and Janet are playing a game in which they take turns removing one or two counters from a pile. Whoever takes the last counter wins. There are 4 counters left, and it is Janet's turn. How many counters should she take?* ** a. 1* *b. 2* *c. It does not matter; she will win.* *d. It does not matter; she will lose.*

Standard 4.5 Mathematical Processes

All students will use mathematical processes of problem solving, communication, connections, reasoning, representations, and technology to solve problems and communicate mathematical ideas.

While no additional big ideas, essential questions, or enduring understandings are listed for this standard, the mathematical processes are imbedded in the content-specific ideas, questions, and understandings delineated for the first four standards. References to the relevant processes can be found above.

4.5 A. Problem Solving

Descriptive Statement: *Problem posing and problem solving involve examining situations that arise in mathematics and other disciplines and in common experiences, describing these situations mathematically, formulating appropriate mathematical questions, and using a variety of strategies to find solutions. Through problem solving, students experience the power and usefulness of mathematics. Problem solving is interwoven throughout the grades to provide a context for learning and applying mathematical ideas.*

Areas of Focus	Comments and Examples
1. Learn mathematics through problem solving, inquiry, and discovery.	**Instructional Focus:** • This CPI is largely an instructional CPI and is assessed within the context of one or more of the content CPIs 4.1 through 4.4.
2. Solve problems that arise in mathematics and in other contexts: Open-ended problems; Non-routine problems; Problems with multiple solutions; Problems that can be solved in several ways.	Assessment of this CPI is within the context of one or more of the content CPIs 4.1 through 4.4.
3. Select and apply a variety of appropriate problem-solving strategies (e.g., "try a simpler problem" or "make a diagram") to solve problems.	Assessment of this CPI is within the context of one or more of the content CPIs 4.1 through 4.4.
4. Pose problems of various types and levels of difficulty.	**Instructional Focus:** • This CPI is largely an instructional CPI and is assessed within the context of one or more of the content CPIs 4.1 through 4.4.
5. Monitor their progress and reflect on the process of their problem solving activity.	Assessment of this CPI is within the context of one or more of the content CPIs 4.1 through 4.4.
6. <u>Distinguish relevant from irrelevant information, and identify missing information.</u>	**Instructional Focus:** • This CPI was added by the State Board of Education on January 9, 2008.. • Assessment of this CPI is within the context of one or more of the content CPIs 4.1 through 4.4.

4.5 B. Communication

Descriptive Statement: *Communication of mathematical ideas involves students' sharing their mathematical understandings in oral and written form with their classmates, teachers, and parents. Such communication helps students clarify and solidify their understanding of mathematics and develop confidence in themselves as mathematics learners. It also enables teachers to better monitor student progress.*

Areas of Focus	Comments and Examples
1. Use communication to organize and clarify mathematical thinking: Reading and writing; Discussion, listening, and questioning.	Assessment of this CPI is within the context of one or more of the content CPIs 4.1 through 4.4.
2. Communicate mathematical thinking coherently and clearly to peers, teachers, and others, both orally and in writing.	Assessment of this CPI is within the context of one or more of the content CPIs 4.1 through 4.4.
3. Analyze and evaluate the mathematical thinking and strategies of others.	Assessment of this CPI is within the context of one or more of the content CPIs 4.1 through 4.4.
4. Use the language of mathematics to express mathematical ideas precisely.	Assessment of this CPI is within the context of one or more of the content CPIs 4.1 through 4.4.

4.5 C. Connections

Descriptive Statement: *Making connections involves seeing relationships between different topics, and drawing on those relationships in future study. This applies within mathematics, so that students can translate readily between fractions and decimals, or between algebra and geometry; to other content areas, so that students understand how mathematics is used in the sciences, the social sciences, and the arts; and to the everyday world, so that students can connect school mathematics to daily life.*

Areas of Focus	Comments and Examples
1. Recognize recurring themes across mathematical domains (e.g., patterns in number, algebra, and geometry).	Assessment of this CPI is within the context of one or more of the content CPIs 4.1 through 4.4.
2. Use connections among mathematical ideas to explain concepts (e.g., two linear equations have a unique solution because the lines they represent intersect at a single point).	Assessment of this CPI is within the context of one or more of the content CPIs 4.1 through 4.4.
3. Recognize that mathematics is used in a variety of contexts outside of mathematics.	**Instructional Focus:** • This CPI is largely an instructional CPI and is assessed within the context of one or more of the content CPIs 4.1 through 4.4.
4. Apply mathematics in practical situations and in other disciplines.	Assessment of this CPI is within the context of one or more of the content CPIs 4.1 through 4.4.
5. Trace the development of mathematical concepts over time and across cultures (cf. world languages and social studies standards).	**Instructional Focus:** • This CPI is largely an instructional CPI and is assessed within the context of one or more of the content CPIs 4.1 through 4.4.
6. Understand how mathematical ideas interconnect and build on one another to produce a coherent whole.	**Instructional Focus:** • This CPI is largely an instructional CPI and is assessed within the context of one or more of the content CPIs 4.1 through 4.4.

4.5 D. Reasoning

Descriptive Statement: *Mathematical reasoning is the critical skill that enables a student to make use of all other mathematical skills. With the development of mathematical reasoning, students recognize that mathematics makes sense and can be understood. They learn how to evaluate situations, select problem-solving strategies, draw logical conclusions, develop and describe solutions, and recognize how those solutions can be applied.*

Areas of Focus	Comments and Examples
1. Recognize that mathematical facts, procedures, and claims must be justified.	**Instructional Focus:** • This CPI is largely an instructional CPI and is assessed within the context of one or more of the content CPIs 4.1 through 4.4.
2. Use reasoning to support their mathematical conclusions and problem solutions.	Assessment of this CPI is within the context of one or more of the content CPIs 4.1 through 4.4.
3. Select and use various types of reasoning and methods of proof.	This may be included in classroom enrichment activities at this grade level, but is more of a focus at secondary grade levels.
4. Rely on reasoning, rather than answer keys, teachers, or peers, to check the correctness of their problem solutions.	Assessment of this CPI is within the context of one or more of the content CPIs 4.1 through 4.4.
5. Make and investigate mathematical conjectures: Counterexamples as a means of disproving conjectures; Verifying conjectures using informal reasoning or proofs.	This may be included in classroom enrichment activities at this grade level, but is more of a focus at higher grade levels.
6. Evaluate examples of mathematical reasoning and determine whether they are valid.	This is more of a focus at secondary grade levels.

4.5 E. Representations

Descriptive Statement: *Representations refers to the use of physical objects, drawings, charts, graphs, and symbols to represent mathematical concepts and problem situations. By using various representations, students will be better able to communicate their thinking and solve problems. Using multiple representations will enrich the problem solver with alternative perspectives on the problem. Historically, people have developed and successfully used manipulatives (concrete representations such as fingers, base ten blocks, geoboards, and algebra tiles) and other representations (such as coordinate systems) to help them understand and develop mathematics.*

Areas of Focus	Comments and Examples
1. Create and use representations to organize, record, and communicate mathematical ideas: Concrete representations (e.g., base-ten blocks or algebra tiles); Pictorial representations (e.g., diagrams, charts, or tables); Symbolic representations (e.g., a formula); Graphical representations (e.g., a line graph).	Assessment of this CPI is within the context of one or more of the content CPIs 4.1 through 4.4.
2. Select, apply, and translate among mathematical representations to solve problems.	Assessment of this CPI is within the context of one or more of the content CPIs 4.1 through 4.4.

3. Use representations to model and interpret physical, social, and mathematical phenomena.	Assessment of this CPI is within the context of one or more of the content CPIs 4.1 through 4.4.

4.5 F. Technology

Descriptive Statement: *Calculators and computers need to be used along with other mathematical tools by students in both instructional and assessment activities. These tools should be used, not to replace mental math and paper-and-pencil computational skills, but to enhance understanding of mathematics and the power to use mathematics. Students should explore both new and familiar concepts with calculators and computers and should also become proficient in using technology as it is used by adults (e.g., for assistance in solving real-world problems).*

Areas of Focus	Comments and Examples
1. Use technology to gather, analyze, and communicate mathematical information.	**Instructional Focus:** • This CPI is largely an instructional CPI and is assessed within the context of one or more of the content CPIs 4.1 through 4.4.
2. Use computer spreadsheets, software, and graphing utilities to organize and display quantitative information.	**Instructional Focus:** • This CPI is largely an instructional CPI and is assessed within the context of one or more of the content CPIs 4.1 through 4.4.
3. Use graphing calculators and computer software to investigate properties of functions and their graphs.	**Instructional Focus:** • This CPI is largely an instructional CPI and is assessed within the context of one or more of the content CPIs 4.1 through 4.4.
4. Use calculators as problem-solving tools (e.g., to explore patterns, to validate solutions).	Assessment of this CPI is within the context of one or more of the content CPIs 4.1 through 4.4.
5. Use computer software to make and verify conjectures about geometric objects.	**Instructional Focus:** • This CPI is largely an instructional CPI and is assessed within the context of one or more of the content CPIs 4.1 through 4.4.
6. Use computer-based laboratory technology for mathematical applications in the sciences.	**Instructional Focus:** • This CPI is largely an instructional CPI and is assessed within the context of one or more of the content CPIs 4.1 through 4.4.