Mathematics Areas of Focus: Grade 8

Mission: *Through mathematics, students communicate, make connections, reason, and represent the world quantitatively in order to pose and solve problems.*

Standard 4.1 Number and Numerical Operations

All students will develop number sense and will perform standard numerical operations and estimations on all types of numbers in a variety of ways.

Big Idea: Numeric reasoning involves fluency and facility with numbers.

4.1.8 A. Number Sense

Descriptive Statement: *Number sense is an intuitive feel for numbers and a common sense approach to using them. It is a comfort with what numbers represent that comes from investigating their characteristics and using them in diverse situations. It involves an understanding of how different types of numbers, such as fractions and decimals, are related to each other, and how each can best be used to describe a particular situation. It subsumes the more traditional category of school mathematics curriculum called numeration and thus includes the important concepts of place value, number base, magnitude, and approximation and estimation.*

Essential Questions	Enduring Understandings
▪ How do mathematical ideas interconnect and build on one another to produce a coherent whole? (4.5C1; 4.5C6)**	▪ One representation may sometimes be more helpful than another; and, used together, multiple representations give a fuller understanding of a problem.
▪ How can we compare and contrast numbers? (4.5A4)**	▪ A quantity can be represented numerically in various ways. Problem solving depends upon choosing wise ways.
▪ How can counting, measuring, or labeling help to make sense of the world around us?	▪ Numeric fluency includes both the understanding of and the ability to appropriately use numbers.

Areas of Focus	Comments and Examples
1. Extend understanding of the number system by constructing meanings for the following (unless otherwise noted, all indicators for grade 8 pertain to these sets of numbers as well): Rational numbers; Percents; Exponents; Roots; Absolute values; Numbers represented in scientific notation.	It is important to note that the sets of numbers specified in this CPI also apply to the other grade 8 mathematics CPIs.
2. Demonstrate a sense of the relative magnitudes of numbers.	**Instructional/Assessment Focus:** • Refers to Rational numbers; Percents; Exponents; Roots; Absolute values; and Numbers represented in scientific notation, as specified in 4.1.8A1. ***Sample Extended Constructed Response (ECR) Item:*** A B <--|---|--|---|--|------|------|---> -2 -1 0 1 2 *Draw on the number line above where the product of A and B would fall.* *Label your point "C"* *Explain your reasoning in choosing this location.* ***Sample Multiple Choice (MC) Item:*** *Which of the operations described below yields the largest number?* ** a. 0.1 divided by 0.2* *b. The sum of 0.1 and 0.2* *c. The product of 0.1 and 0.2* *d. 0.1 raised to the second power*
3. Understand and use ratios, **rates,** proportions, and percents (including percents greater than 100 and less than 1) in a variety of situations.	**Instructional/Assessment Focus:** • Much of this content is an area of focus in grade 7 and may be assessed at a higher level of understanding in grade 8. • The word "rates" was added to this CPI by the State Board of Education on January 9, 2008. This is an area of focus in grade 8 and should be linked to the concept of slope (4.3.8B1).
4. Compare and order numbers of all named types.	**Instructional/Assessment Focus:** • Refers to Rational numbers; Percents; Exponents; Roots; Absolute values; Numbers represented in scientific notation, as specified in 4.1.8A1
5. Use whole numbers, fractions, decimals, and percents to represent equivalent forms of the same number.	This is an area of focus in grade 7 and may be assessed at a higher level of understanding in grade 8.

6. Recognize that repeating decimals correspond to fractions and determine their fractional equivalents: 5/7 = 0.714285714285... = $0.\overline{714285}$	Assessment of this CPI is generally within the context of one or more of the other content CPIs.
7. Construct meanings for common irrational numbers, such as pi and the square root of 2.	"Construct meanings" means, for assessment purposes, "give concrete representations." ***Sample ECR Item:*** *With only a ruler and a pencil, explain how you could approximate the value of $\sqrt{2}$.* ***Sample ECR Item:*** *With only a DVD and a piece of string, explain how you could approximate the value of π.*

4.1.8 B. Numerical Operations

Descriptive Statement: *Numerical Operations are an essential part of the mathematics curriculum, especially in the elementary grades. Students must be able to select and apply various computational methods, including mental math, pencil-and-paper techniques, and the use of calculators. Students must understand how to add, subtract, multiply, and divide whole numbers, fractions, decimals, and other kinds of numbers. With the availability of calculators that perform these operations quickly and accurately, the instructional emphasis now is on understanding the meanings and uses of these operations, and on estimation and mental skills, rather than solely on the development of paper-and-pencil proficiency.*

Essential Questions	Enduring Understandings
▪ What makes a computational strategy both effective and efficient? (4.5D1)**	▪ Computational fluency includes understanding the meaning and the appropriate use of numerical operations.
▪ How do operations affect numbers?	▪ The magnitude of numbers affects the outcome of operations on them.
▪ How do mathematical representations reflect the needs of society across cultures? (An essential question with broad applicability across multiple standards) (4.5C5)**	▪ In many cases, there are multiple algorithms for finding a mathematical solution, and those algorithms are frequently associated with different cultures.

Areas of Focus	Comments and Examples
1. Use and explain procedures for performing calculations involving addition, subtraction, multiplication, division, and exponentiation with integers and all number types named above with: Pencil-and-paper; Mental math; Calculator.	
2. Use exponentiation to find whole number powers of numbers.	This is an area of focus in grade 7 and may be assessed at a higher level of understanding in grade 8.
3. Find square and cube roots of numbers and understand the inverse nature of powers and roots.	***Sample MC Item:*** *Pat has 1296 one-inch square tiles. Which of the following are the dimensions of the largest square table top Pat could cover with the tiles?* *a. 324 in. x 324 in. b. 9 ft x 9 ft * c. 1 yd x 1 yd d. 36 m x 36 m*
4. Solve problems involving proportions and percents.	This includes CPIs 4.5A2, 4.5B1, 4.5D2, and 4.5E2.**
5. Understand and apply the standard algebraic order of operations, including appropriate use of parentheses.	This is an area of focus in grade 7 and may be assessed at a higher level of understanding in grade 8.

4.1.8 C. Estimation

Descriptive Statement: *Estimation is a process that is used constantly by mathematically capable adults, and one that can be easily mastered by children. It involves an educated guess about a quantity or an intelligent prediction of the outcome of a computation. The growing use of calculators makes it more important than ever that students know when a computed answer is reasonable; the best way to make that determination is through the use of strong estimation skills. Equally important is an awareness of the many situations in which an approximate answer is as good as, or even preferable to, an exact one. Students can learn to make these judgments and use mathematics more powerfully as a result.*

Essential Questions	Enduring Understandings
▪ How can we decide when to use an exact answer and when to use an estimate?	▪ Context is critical when using estimation.

Areas of Focus	Comments and Examples
1. Estimate square and cube roots of numbers.	
2. Use equivalent representations of numbers such as fractions, decimals, and percents to facilitate estimation.	This is an area of focus in grade 7 and may be assessed at a higher level of understanding in grade 8.
3. Recognize the limitations of estimation and assess the amount of error resulting from estimation.	

Standard 4.2 Geometry and Measurement

All students will develop spatial sense and the ability to use geometric properties, relationships, and measurement to model, describe and analyze phenomena.

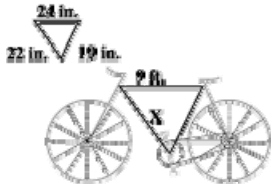
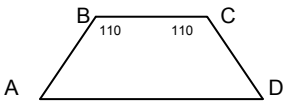
Big Idea *Geometry*: Spatial sense and geometric relationships are a means to solve problems and make sense of a variety of phenomena.

Big Idea *Measurement*: Measurement is a tool to quantify a variety of phenomena.

4.2.8 A. Geometric Properties

Descriptive Statement: *This includes identifying, describing and classifying standard geometric object, describing and comparing properties of geometric objects, making conjectures concerning them, and using reasoning and proof to verify or refute conjectures and theorems. Also included here are such concepts as symmetry, congruence, and similarity.*

Essential Questions	Enduring Understandings
▪ How can spatial relationships be described by careful use of geometric language? ▪ How do geometric relationships help to solve problems and/or make sense of phenomena?	▪ Geometric properties can be used to construct geometric figures. (4.5D1; 4.5D2; 4.5E3)** ▪ Geometric relationships provide a means to make sense of a variety of phenomena.

Areas of Focus	Comments and Examples
1. Understand and apply concepts involving lines, angles, and planes: Complementary and supplementary angles; Vertical angles; Bisectors and perpendicular bisectors; Parallel, perpendicular, and intersecting planes; Intersection of plane with cube, cylinder, cone, and sphere.	
2. Understand and apply the Pythagorean Theorem.	
3. Understand and apply properties of polygons: Quadrilaterals, including squares, rectangles, parallelograms, trapezoids, rhombi; Regular polygons; Sum of measures of interior angles of a polygon; Which polygons can be used alone to generate a tessellation and why.	
4. Understand and apply the concept of similarity: Using proportions to find missing measures; Scale drawings; Models of 3D objects.	***Sample ECR Item:*** *A billboard designer must enlarge this picture of a bicycle to fit on a large outdoor sign.*  *The actual dimensions of the seat-pedal-handle bar triangle are shown below. The bar connecting the seat with the handle bars, for example, is 24 inches long. The corresponding part on the billboard would be 9 feet long. Find the missing billboard length, X, as shown. (Answer: x = 8.25 ft. or 8 ft. 3 in. or 99 in.)* 
5. Use logic and reasoning to make and support conjectures about geometric objects.	***Sample ECR Item:*** *The stage in the auditorium at Park Middle School is shaped like a quadrilateral, with each of two angles measuring 110°, as shown in the diagram below.*  *In the drawing, ∠A is congruent to ∠D. What is the degree measure of ∠A? Explain your reasoning to support your answer.* Assessment of this CPI is generally within the context of one or more of the other content CPIs, as illustrated by the sample linked to CPI 4.2.8A3.

6. <u>Perform basic geometric constructions using a variety of methods (e.g., straightedge and compass, patty/tracing paper, or technology).</u> • **<u>Congruent angles or line segments</u>** • **<u>Midpoint of a line segment</u>**	This CPI was added by the State Board of Education on January 9, 2008 and is an area of focus in grade 8.
7. <u>Create two-dimensional representations (e.g., nets or projective views) for the surfaces of three-dimensional objects.</u>	This CPI was added by the State Board of Education on January 9, 2008 and is an area of focus in grade 8.

4.2.8 B. Transforming Shapes

Descriptive Statement: *This includes identifying, describing and classifying standard geometric object, describing and comparing properties of geometric objects, making conjectures concerning them, and using reasoning and proof to verify or refute conjectures and theorems. Also included here are such concepts as symmetry, congruence, and similarity.*

Essential Questions	Enduring Understandings
▪ What situations can be analyzed using transformations and symmetries? (4.5E1; 4.5E2; 4.5E3)**	▪ Shape and area can be conserved during mathematical transformations.
Areas of Focus	**Comments and Examples**
1. Understand and apply transformations: Finding the image, given the pre-image, and vice-versa; Sequence of transformations needed to map one figure onto another; Reflections, rotations, and translations result in images congruent to the pre-image; Dilations (stretching/shrinking) result in images similar to the pre-image.	This is an area of focus in grade 7 and may be assessed at a higher level of understanding in grade 8.
2. Use iterative procedures to generate geometric patterns: Fractals (e.g., the Koch Snowflake); Self-similarity; Construction of initial stages; Patterns in successive stages (e.g., number of triangles in each stage of Sierpinski's Triangle).	

4.2.8 C. Coordinate Geometry

Descriptive Statement: *Coordinate geometry provides an important connection between geometry and algebra. It facilitates the visualization of algebraic relationships, as well as an analytical understanding of geometry.*

Essential Questions	Enduring Understandings
▪ How can geometric/algebraic relationships best be represented and verified? (4.5C2; 4.5D2; 4.5E1; 4.5E2; 4.5F5)**	▪ Reasoning and/or proof can be used to verify or refute conjectures or theorems in geometry (4.5D1; 4.5D3; 4.5D4; 4.5D5; 4.5F5)** ▪ Coordinate geometry can be used to represent and verify geometric/algebraic relationships.
Areas of Focus	**Comments and Examples**
1. Use coordinates in four quadrants to represent geometric concepts.	
2. Use a coordinate grid to model and quantify transformations (e.g., translate right 4 units).	

4.2.8 D. Units Of Measurement

Descriptive Statement: *Measurement helps describe our world using numbers. An understanding of how we attach numbers to real-world phenomena, familiarity with common measurement units (e.g., inches, liters, and miles per hour), and a practical knowledge of measurement tools and techniques are critical for students' understanding of the world around them.*

Essential Questions	Enduring Understandings
▪ How can measurements be used to solve problems? (4.5A6)**	▪ Everyday objects have a variety of attributes, each of which can be measured in many ways. ▪ What we measure affects how we measure it. (4.5A4; 4.5A6)** ▪ Measurements can be used to describe, compare, and make sense of phenomena.
Areas of Focus	**Comments and Examples**
1. Solve problems requiring calculations that involve different units of measurement within a measurement system (e.g., 4'3" plus 7'10" equals 12'1").	***Sample ECR Item:*** *You are purchasing a wallpaper border that will go around the top of a room. The room measures 8 feet 9 inches by 13 feet 8 inches. If the border is sold by the yard, how many whole yards will you need to buy? Explain your reasoning to support your answer.* ***Sample Short Constructed Response (SCR) Item:*** *A tire is 25 inches in diameter. How many times will it turn in traveling a mile?*

2. Use approximate equivalents between standard and metric systems to estimate measurements (e.g., kilometers is about 3 miles).	Assessment of this CPI is generally within the context of one or more of the other content CPIs.
3. Recognize that the degree of precision needed in calculations depends on how the results will be used and the instruments used to generate the measurements.	
4. Select and use appropriate units and tools to measure quantities to the degree of precision needed in a particular problem-solving situation.	
5. Recognize that all measurements of continuous quantities are approximations.	Assessment of this CPI is generally within the context of one or more of the other content CPIs.
6. Solve problems that involve compound measurement units, such as speed (miles per hour), air pressure (pounds per square inch), and population density (persons per square mile).	***Sample MC Item:*** *Which object listed in the table below has the greatest density?* **Object** / **Mass of Object** / **Volume of Object** A / 11.0 grams / 24 cubic centimeters B / 11.0 grams / 12 cubic centimeters C / 5.5 grams / 4 cubic centimeters D / 5.5 grams / 11 cubic centimeters *a. A b. B * c. C d. D*

4.2.8 E. Measuring Geometric Objects

Descriptive Statement: *This area focuses on applying the knowledge and understandings of units of measurement in order to actually perform measurement. While students will eventually apply formulas, it is important they develop and apply strategies that derive from their understanding of the attributes. In addition to measuring objects directly, students apply indirect measurement skills, using, for example, similar triangles and trigonometry.*

Essential Questions	Enduring Understandings
▪ How can measurements be used to solve problems? (4.5A6)**	▪ Everyday objects have a variety of attributes, each of which can be measured in many ways. ▪ What we measure affects how we measure it. (4.5A4; 4.5A6)** ▪ Measurements can be used to describe, compare, and make sense of phenomena.

Areas of Focus	Comments and Examples
1. Develop and apply strategies for finding perimeter and area: Geometric figures made by combining triangles, rectangles and circles or parts of circles; Estimation of area using grids of various sizes; Impact of a dilation on the perimeter and area of a 2-dimensional figure; Impact of a dilation on the perimeter and area of a 2-dimensional figure.	
2. Recognize that the volume of a pyramid or cone is one-third of the volume of the prism or cylinder with the same base and height (e.g., use rice to compare volumes of figures with same base and height).	Assessment of this CPI is generally within the context of CPI 4.2.8E3.
3. Develop and apply strategies and formulas for finding the surface area and volume of a three-dimensional figure: Volume —prism, cone, pyramid; Surface area —prism (triangular or rectangular base), pyramid (triangular or rectangular base); Impact of a dilation on the surface area and volume of a three-dimensional figure.	
4. Use formulas to find the volume and surface area of a sphere.	

Standard 4.3 Patterns and Algebra

All students will represent and analyze relationships among variable quantities and solve problems involving patterns, functions, and algebraic concepts and processes.

Big Idea: Algebra provides language through which we communicate the patterns in mathematics.

4.3.8 A. Patterns

Descriptive Statement: *Algebra provides the language through which we communicate the patterns in mathematics. From the earliest age, students should be encouraged to investigate the patterns that they find in numbers, shapes, and expressions, and by doing so, to make mathematical discoveries. They should have opportunities to analyze, extend, and create a variety of patterns and to use pattern-based thinking to understand and represent mathematical and other real-world phenomena.*

Essential Questions	Enduring Understandings
▪ How can change be best represented mathematically? (4.5C1; 4.5F1; 4.5F2; 4.5F3; 4.5F4)** ▪ How can patterns, relations, and functions be used as tools to best describe and help explain real-life situations? (4.5C1)**	▪ The symbolic language of algebra is used to communicate and generalize the patterns in mathematics. ▪ Algebraic representation can be used to generalize patterns and relationships.

Areas of Focus	Comments and Examples
1. Recognize, describe, extend, and create patterns involving whole numbers, rational numbers, and integers: Descriptions using tables, verbal and symbolic rules, graphs, simple equations or expressions; Finite and infinite sequences; Arithmetic sequences (i.e., sequences generated by repeated addition of a fixed number, positive or negative); Geometric sequences (i.e., sequences generated by repeated multiplication by a fixed positive ratio, greater than 1 or less than 1); Generating sequences by using calculators to repeatedly apply a formula.	

4.3.8 B. Functions & Relationships

Descriptive Statement: *The function concept is one of the most fundamental unifying ideas of modern mathematics. Students begin their study of functions in the primary grades, as they observe and study patterns. As students grow and their ability to abstract matures, students form rules, display information in a table or chart, and write equations which express the relationships they have observed. In high school, they use the more formal language of algebra to describe these relationships.*

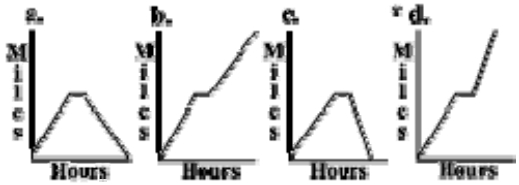
Essential Questions	Enduring Understandings
▪ How are patterns of change related to the behavior of functions? (4.5F1; 4.5F2; 4.5F3; 4.5F4)**	▪ Patterns and relationships can be represented graphically, numerically, symbolically, or verbally. (4.5E1)**

Areas of Focus	Comments and Examples
1. Graph functions, and understand and describe their general behavior: Equations involving two variables; Equations involving two variables; Rates of change (informal notion of slope).	
2. Recognize and describe the difference between linear and exponential growth, using tables, graphs, and equations.	

4.3.8 C. Modeling

Descriptive Statement: *Algebra is used to model real situations and answer questions about them. This use of algebra requires the ability to represent data in tables, pictures, graphs, equations or inequalities, and rules. Modeling ranges from writing simple number sentences to help solve story problems in the primary grades to using functions to describe the relationship between two variables, such as the height of a pitched ball over time. Modeling also includes some of the conceptual building blocks of calculus, such as how quantities change over time and what happens in the long run (limits).*

Essential Questions	Enduring Understandings
▪ How can mathematical models be used to describe physical relationships? (4.5E2)** ▪ How physical models be used to clarify mathematical relationships? (4.5E3)**	▪ Mathematical models can be used to describe and quantify physical relationships. (4.5E2)** ▪ Physical models can be used to clarify mathematical relationships. (4.5E3)**

Areas of Focus	Comments and Examples
1. Analyze functional relationships to explain how a change in one quantity can result in a change in another, using pictures, graphs, charts, and equations.	***Sample MC Item:*** *The Viking Hiking Club hikes a part of the Appalachian Trail once a year. On one occasion they hiked up the mountain at a constant rate until they reached Sunfish Pond. After a short rest, they hiked back down the mountain at a slightly faster constant rate. Which graph shows the relationship between the distance they traveled and the time it took them?* 
2. Use patterns, relations, symbolic algebra, and linear functions to model situations: Using concrete materials (manipulatives), tables, graphs, verbal rules, algebraic expressions/ equations/ inequalities; Growth situations, such as population growth and compound interest, using recursive (e.g., NOW-NEXT) formulas (cf. science standards and social studies standards).	

4.3.8 D. Procedures

Descriptive Statement: *Techniques for manipulating algebraic expressions - procedures - remain important, especially for students who may continue their study of mathematics in a calculus program. Utilization of algebraic procedures includes understanding and applying properties of numbers and operations, using symbols and variables appropriately, working with expressions, equations, and inequalities, and solving equations and inequalities.*

Essential Questions	Enduring Understandings
▪ What makes an algebraic algorithm both effective and efficient? (4.5D1)**	▪ Algebraic and numeric procedures are interconnected and build on one another to produce a coherent whole. ▪ Reasoning and/or proof can be used to verify or refute conjectures or theorems in algebra. (4.5D1; 4.5D3; 4.5D4; 4.5D5)**

Areas of Focus	Comments and Examples
1. Use graphing techniques on a number line: Absolute value; Arithmetic operations represented by vectors (arrows) (e.g., "-3 + 6" is "left 3, right 6")	This is an area of focus in grade 7 and may be assessed at a higher level of understanding in grade 8.
2. Solve simple linear equations informally, graphically, and using formal algebraic methods: ▪ **Multi-step, integer coefficients only (although answers may not be integers)** ▪ **<u>Simple literal equations (e.g., A = lw)</u>** ▪ **Using paper-and-pencil, calculators, graphing calculators, spreadsheets, and other technology.**	The second bullet of this CPI was added by the State Board of Education on January 9, 2008.
3. Solve simple linear inequalities.	
4. Create, evaluate, and simplify algebraic expressions involving variables: Order of operations, including appropriate use of parentheses; Distributive property; Substitution of a number for a variable; Translation of a verbal phrase or sentence into an algebraic expression, equation, or inequality, and vice versa.	**Instructional/Assessment Focus:** • "Create" implies within a problem-solving situation, consistent with 4.5A2. ***Sample SCR Item:*** *The amount A that principal P will be worth after t years at interest rate r, compounded annually, is given by this formula:* $A = P(1 + r)^t$ *Suppose $4,000 principal is invested at 6% interest compounded annually for five years. How much money would the investment yield after 5 years?* (Answer: $5,532.90)
5. Understand and apply the properties of operations, numbers, equations, and inequalities: Additive inverse; Multiplicative inverse; Addition and multiplication properties of equality; Addition and multiplication properties of inequalities.	***Sample MC Item:*** *If X > 0 and Y < 0, what must be true about the value of the expression – X – Y ?* ** a. It is sometimes positive* *b. It is always negative* *c. It is never negative* *d. It is never zero*

Standard 4.4 Data Analysis, Probability, and Discrete Mathematics

All students will develop an understanding of the concepts and techniques of data analysis, probability, and discrete mathematics, and will use them to model situations, solve problems, and analyze and draw appropriate inferences from data.

Big Idea *Data Analysis***:** Reading, understanding, interpreting, and communicating data are critical in modeling a variety of real-world situations, drawing appropriate inferences, making informed decisions, and justifying those decisions.

Big Idea *Probability***:** Probability quantifies the likelihood that something will happen and enables us to make predictions and informed decisions.

Big Idea *Discrete Mathematics***:** Discrete mathematics consists of tools and strategies for representing, organizing, and interpreting non-continuous data.

4.4.8 A. Data Analysis

Descriptive Statement: *In today's information-based world, students need to be able to read, understand, and interpret data in order to make informed decisions. In the early grades, students should be involved in collecting and organizing data, and in presenting it using tables, charts, and graphs. As they progress, they should gather data using sampling, and should increasingly be expected to analyze and make inferences from data, as well as to analyze data and inferences made by others.*

Essential Questions	Enduring Understandings
▪ How can the collection, organization, interpretation, and display of data be used to answer questions? (4.5A4; 4.5A6; 4.5E1; 4.5E2; 4.5F1; 4.5F6)**	▪ The message conveyed by the data depends on how the data is collected, represented, and summarized. (4.5A6; 4.5D6; 4.5E1; 4.5E2; 4.5E3)** ▪ The results of a statistical investigation can be used to support or refute an argument. (4.5D1; 4.5D3; 4.5D5; 4.5E2; 4.5E3; 4.5F6)**

Areas of Focus	Comments and Examples
1. Select and use appropriate representations for sets of data, and measures of central tendency (mean, median, and mode): Type of display most appropriate for given data; Box-and-whisker plot, upper quartile, lower quartile; Scatter plot; Calculators and computer used to record and process information; Finding the median and mean (weighted average) using frequency data; Effect of additional data on measures of central tendency.	***Sample MC Item:*** *Student Age* — *Number of Students* *12* — *1* *13* — *4* *14* — *25* *The table above shows the ages of the students in Elaine's class. To the nearest tenth of a year, what is the mean of the 30 students' ages?* *a. 13.0 b. 13.4 * c. 13.8 d. 14.0*
2. Make inferences and formulate and evaluate arguments based on displays and analysis of data sets.	The word "sets" was added to this CPI by the State Board of Education on January 9, 2008.
3. Estimate lines of best fit and use them to interpolate within the range of the data.	
4. Use surveys and sampling techniques to generate data and draw conclusions about large groups.	**Instructional Focus:** This CPI is largely an instructional CPI. Assessment of this CPI is generally within the context of one or more of the other content CPIs.

4.4.8 B. Probability

Descriptive Statement: *Students need to understand the fundamental concepts of probability so that they can interpret weather forecasts, avoid unfair games of chance, and make informed decisions about medical treatments whose success rate is provided in terms of percentages. They should regularly be engaged in predicting and determining probabilities, often based on experiments (like flipping a coin 100 times), but eventually based on theoretical discussions of probability that make use of systematic counting strategies. High school students should use probability models and solve problems involving compound events and sampling.*

Essential Questions	Enduring Understandings
▪ How can experimental and theoretical probabilities be used to make predictions or draw conclusions? (4.5D5; 4.5D6)**	▪ Experimental results tend to approach theoretical probabilities after a large number of trials.

Areas of Focus	Comments and Examples
1. Interpret probabilities as ratios, percents, and decimals.	**Instructional/Assessment Focus:** • "Interpret" here includes recognizing equivalent forms for expressing a probability. Assessment of this CPI is generally within the context of one or more of the other content CPIs.

2. Determine probabilities of compound events.	***Sample MC Item:*** *Jeremy has a fair coin and a number cube with the sides labeled one through six. What is the probability of getting both a head on a toss of the coin and a four on a roll of the number cube?* *a. 2/3 b. 1/2 c. 1/3 * d. 1/12*
3. Explore the probabilities of conditional events (e.g., if there are seven marbles in a bag, three red and four green, what is the probability that two marbles picked from the bag, without replacement, are both red).	Content should be introduced at this grade level, but mastery of the content should not be assessed in statewide assessment at this grade level.
4. Model situations involving probability with simulations (using spinners, dice, calculators and computers) and theoretical models: Frequency, relative frequency.	**Instructional Focus:** • This CPI is largely an instructional CPI. Assessment of this CPI is generally within the context of one or more of the other content CPIs. ***Sample Classroom Performance Task:*** *Design a spinner that has the following probabilities:* *P(red) = 3/8 P(blue) = 25 %* *P(yellow) = 12 ½ % P(white) = remaining section* *Design means to draw your spinner and label each section with its appropriate color and probability.* *• Is this a fair spinner? Why or why not? Explain your reasoning.* *• Devise a fair game using this spinner. Describe your game.*
5. Estimate probabilities and make predictions based on experimental and theoretical probabilities.	This is an area of focus in grade 7 and may be assessed at a higher level of understanding in grade 8.
6. Play and analyze probability-based games, and discuss the concepts of fairness and expected value.	This is an area of focus in grade 7 and may be assessed at a higher level of understanding in grade 8. **Instructional/Assessment Focus:** • Assessment will focus on analysis of the probabilities, more than the playing of the games. The actual playing of games may appropriately receive additional attention during instruction. "Discuss" here means "explain."

4.4.8 C. Discrete Mathematics - Systematic Listing And Counting

Descriptive Statement: *Development of strategies for listing and counting can progress through all grade levels, with middle and high school students using the strategies to solve problems in probability. Primary students, for example, might find all outfits that can be worn using two coats and three hats; middle school students might systematically list and count the number of routes from one site on a map to another; and high school students might determine the number of three-person delegations that can be selected from their class to visit the mayor.*

Essential Questions	Enduring Understandings
▪ How can attributes be used to classify data/objects? ▪ What is the best way to solve this? What counting strategy works best here?	▪ Grouping by attributes (classification) can be used to answer mathematical questions. (4.5E1; 4.5E3)** ▪ Algorithms can effectively and efficiently be used to quantify and interpret discrete information.

Areas of Focus	Comments and Examples
1. Apply the multiplication principle of counting: Permutations: ordered situations with replacement (e.g., number of possible license plates) vs. ordered situations without replacement (e.g., number of possible slates of 3 class officers from a 23 student class); Factorial notation; Concept of combinations (e.g., number of possible delegations of 3 out of 23 students).	
2. Explore counting problems involving Venn diagrams with three attributes (e.g., there are 15, 20, and 25 students respectively in the chess club, the debating team, and the engineering society; how many different students belong to the three clubs if there are 6 students in chess and debating, 7 students in chess and engineering, 8 students in debating and engineering, and 2 students in all three?).	**Instructional Focus:** • This content should be introduced at this grade level, but mastery of the content is not assessed in statewide assessment at this grade level.
3. Apply techniques of systematic listing, counting, and reasoning in a variety of different contexts.	This is an area of focus in grade 7 and may be assessed at a higher level of understanding in grade 8.

4.4.8 D. Discrete Mathematics - Vertex-Edge Graphs And Algorithms

Descriptive Statement: *Vertex-edge graphs, consisting of dots (vertices) and lines joining them (edges), can be used to represent and solve problems based on real-world situations. Students should learn to follow and devise lists of instructions, called "algorithms," and use algorithmic thinking to find the best solution to problems like those involving vertex-edge graphs, but also to solve other problems.*

Essential Questions	Enduring Understandings
▪ How can visual tools such as networks (vertex-edge graphs) be used to answer questions? (4.5E1; 4.5E3)** ▪ How can algorithmic thinking be used to solve problems?	▪ Optimization is finding the best solution within given constraints. ▪ Algorithms can effectively and efficiently be used to quantify and interpret discrete information.

Areas of Focus	Comments and Examples
1. Use vertex-edge graphs and algorithmic thinking to represent and find solutions to practical problems: Finding the shortest network connecting specified sites; Finding a minimal route that includes every street (e.g., for trash pick-up); Finding the shortest route on a map from one site to another; Finding the shortest circuit on a map that makes a tour of specified sites; Limitations of computers (e.g., the number of routes for a delivery truck visiting *n* sites is *n*!, so finding the shortest circuit by examining all circuits would overwhelm the capacity of any computer, now or in the future, even if n is less than 100).	***Sample ECR Item:*** *Five classmates greet each other with a handshake so that each person shakes the hand of every other person once and only once.* • *Graph the network of handshakes using the vertices of the graph to represent each person, and the edges to represent the handshakes.* • *How many distinct handshakes were made?* • *If 2 more classmates join the group, and follow the same rule for handshaking, how many handshakes would then be possible?* • *Explain clearly how you arrived at this answer.*

Standard 4.5 Mathematical Processes

All students will use mathematical processes of problem solving, communication, connections, reasoning, representations, and technology to solve problems and communicate mathematical ideas.

While no additional big ideas, essential questions, or enduring understandings are listed for this standard, the mathematical processes are imbedded in the content-specific ideas, questions, and understandings delineated for the first four standards. References to the relevant processes can be found above.

4.5 A. Problem Solving

Descriptive Statement: *Problem posing and problem solving involve examining situations that arise in mathematics and other disciplines and in common experiences, describing these situations mathematically, formulating appropriate mathematical questions, and using a variety of strategies to find solutions. Through problem solving, students experience the power and usefulness of mathematics. Problem solving is interwoven throughout the grades to provide a context for learning and applying mathematical ideas.*

Areas of Focus	Comments and Examples
1. Learn mathematics through problem solving, inquiry, and discovery.	**Instructional Focus:** • This CPI is largely an instructional CPI and is assessed within the context of one or more of the content CPIs 4.1 through 4.4.
2. Solve problems that arise in mathematics and in other contexts: Open-ended problems; Non-routine problems; Problems with multiple solutions; Problems that can be solved in several ways.	Assessment of this CPI is within the context of one or more of the content CPIs 4.1 through 4.4.
3. Select and apply a variety of appropriate problem-solving strategies (e.g., "try a simpler problem" or "make a diagram") to solve problems.	Assessment of this CPI is within the context of one or more of the content CPIs 4.1 through 4.4.
4. Pose problems of various types and levels of difficulty.	**Instructional Focus:** • This CPI is largely an instructional CPI and is assessed within the context of one or more of the content CPIs 4.1 through 4.4.
5. Monitor their progress and reflect on the process of their problem solving activity.	Assessment of this CPI is within the context of one or more of the content CPIs 4.1 through 4.4.
6. <u>Distinguish relevant from irrelevant information, and identify missing information.</u>	**Instructional Focus:** • This CPI was added by the State Board of Education on January 9, 2008.. • Assessment of this CPI is within the context of one or more of the content CPIs 4.1 through 4.4.

4.5 B. Communication

Descriptive Statement: *Communication of mathematical ideas involves students' sharing their mathematical understandings in oral and written form with their classmates, teachers, and parents. Such communication helps students clarify and solidify their understanding of mathematics and develop confidence in themselves as mathematics learners. It also enables teachers to better monitor student progress.*

Areas of Focus	Comments and Examples
1. Use communication to organize and clarify mathematical thinking: Reading and writing; Discussion, listening, and questioning.	Assessment of this CPI is within the context of one or more of the content CPIs 4.1 through 4.4.
2. Communicate mathematical thinking coherently and clearly to peers, teachers, and others, both orally and in writing.	Assessment of this CPI is within the context of one or more of the content CPIs 4.1 through 4.4.
3. Analyze and evaluate the mathematical thinking and strategies of others.	Assessment of this CPI is within the context of one or more of the content CPIs 4.1 through 4.4.
4. Use the language of mathematics to express mathematical ideas precisely.	Assessment of this CPI is within the context of one or more of the content CPIs 4.1 through 4.4.

4.5 C. Connections

Descriptive Statement: *Making connections involves seeing relationships between different topics, and drawing on those relationships in future study. This applies within mathematics, so that students can translate readily between fractions and decimals, or between algebra and geometry; to other content areas, so that students understand how mathematics is used in the sciences, the social sciences, and the arts; and to the everyday world, so that students can connect school mathematics to daily life.*

Areas of Focus	Comments and Examples
1. Recognize recurring themes across mathematical domains (e.g., patterns in number, algebra, and geometry).	Assessment of this CPI is within the context of one or more of the content CPIs 4.1 through 4.4.
2. Use connections among mathematical ideas to explain concepts (e.g., two linear equations have a unique solution because the lines they represent intersect at a single point).	Assessment of this CPI is within the context of one or more of the content CPIs 4.1 through 4.4.
3. Recognize that mathematics is used in a variety of contexts outside of mathematics.	**Instructional Focus:** • This CPI is largely an instructional CPI and is assessed within the context of one or more of the content CPIs 4.1 through 4.4.
4. Apply mathematics in practical situations and in other disciplines.	Assessment of this CPI is within the context of one or more of the content CPIs 4.1 through 4.4.
5. Trace the development of mathematical concepts over time and across cultures (cf. world languages and social studies standards).	**Instructional Focus:** • This CPI is largely an instructional CPI and is assessed within the context of one or more of the content CPIs 4.1 through 4.4.
6. Understand how mathematical ideas interconnect and build on one another to produce a coherent whole.	**Instructional Focus:** • This CPI is largely an instructional CPI and is assessed within the context of one or more of the content CPIs 4.1 through 4.4.

4.5 D. Reasoning

Descriptive Statement: *Mathematical reasoning is the critical skill that enables a student to make use of all other mathematical skills. With the development of mathematical reasoning, students recognize that mathematics makes sense and can be understood. They learn how to evaluate situations, select problem-solving strategies, draw logical conclusions, develop and describe solutions, and recognize how those solutions can be applied.*

Areas of Focus	Comments and Examples
1. Recognize that mathematical facts, procedures, and claims must be justified.	**Instructional Focus:** • This CPI is largely an instructional CPI and is assessed within the context of one or more of the content CPIs 4.1 through 4.4.
2. Use reasoning to support their mathematical conclusions and problem solutions.	Assessment of this CPI is within the context of one or more of the content CPIs 4.1 through 4.4.
3. Select and use various types of reasoning and methods of proof.	This may be included in classroom enrichment activities at this grade level, but is more of a focus at secondary grade levels.
4. Rely on reasoning, rather than answer keys, teachers, or peers, to check the correctness of their problem solutions.	Assessment of this CPI is within the context of one or more of the content CPIs 4.1 through 4.4.
5. Make and investigate mathematical conjectures: Counterexamples as a means of disproving conjectures; Verifying conjectures using informal reasoning or proofs.	Assessment of this CPI is within the context of one or more of the content CPIs 4.1 through 4.4.
6. Evaluate examples of mathematical reasoning and determine whether they are valid.	This may be included in classroom enrichment activities at this grade level, but is more of a focus at secondary grade levels.

4.5 E. Representations	
Descriptive Statement: *Representations refers to the use of physical objects, drawings, charts, graphs, and symbols to represent mathematical concepts and problem situations. By using various representations, students will be better able to communicate their thinking and solve problems. Using multiple representations will enrich the problem solver with alternative perspectives on the problem. Historically, people have developed and successfully used manipulatives (concrete representations such as fingers, base ten blocks, geoboards, and algebra tiles) and other representations (such as coordinate systems) to help them understand and develop mathematics.*	
Areas of Focus	**Comments and Examples**
1. Create and use representations to organize, record, and communicate mathematical ideas: Concrete representations (e.g., base-ten blocks or algebra tiles); Pictorial representations (e.g., diagrams, charts, or tables); Symbolic representations (e.g., a formula); Graphical representations (e.g., a line graph).	Assessment of this CPI is within the context of one or more of the content CPIs 4.1 through 4.4.
2. Select, apply, and translate among mathematical representations to solve problems.	Assessment of this CPI is within the context of one or more of the content CPIs 4.1 through 4.4.
3. Use representations to model and interpret physical, social, and mathematical phenomena.	Assessment of this CPI is within the context of one or more of the content CPIs 4.1 through 4.4.

4.5 F. Technology	
Descriptive Statement: *Calculators and computers need to be used along with other mathematical tools by students in both instructional and assessment activities. These tools should be used, not to replace mental math and paper-and-pencil computational skills, but to enhance understanding of mathematics and the power to use mathematics. Students should explore both new and familiar concepts with calculators and computers and should also become proficient in using technology as it is used by adults (e.g., for assistance in solving real-world problems).*	
Areas of Focus	**Comments and Examples**
1. Use technology to gather, analyze, and communicate mathematical information.	**Instructional Focus:** • This CPI is largely an instructional CPI and is assessed within the context of one or more of the content CPIs 4.1 through 4.4.
2. Use computer spreadsheets, software, and graphing utilities to organize and display quantitative information.	**Instructional Focus:** • This CPI is largely an instructional CPI and is assessed within the context of one or more of the content CPIs 4.1 through 4.4.
3. Use graphing calculators and computer software to investigate properties of functions and their graphs.	**Instructional Focus:** • This CPI is largely an instructional CPI and is assessed within the context of one or more of the content CPIs 4.1 through 4.4.
4. Use calculators as problem-solving tools (e.g., to explore patterns, to validate solutions).	Assessment of this CPI is within the context of one or more of the content CPIs 4.1 through 4.4.
5. Use computer software to make and verify conjectures about geometric objects.	**Instructional Focus:** • This CPI is largely an instructional CPI and is assessed within the context of one or more of the content CPIs 4.1 through 4.4.
6. Use computer-based laboratory technology for mathematical applications in the sciences.	**Instructional Focus:** • This CPI is largely an instructional CPI and is assessed within the context of one or more of the content CPIs 4.1 through 4.4.